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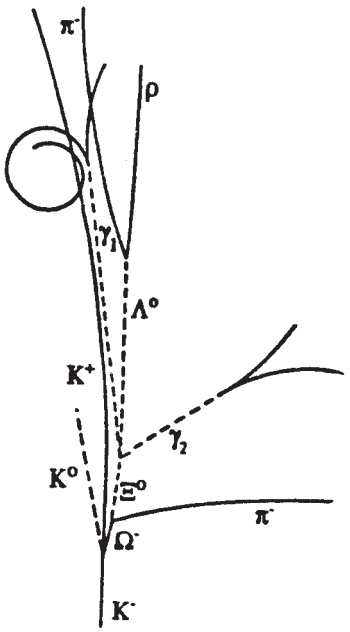

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MODELING THE ELECTRON AS A STABLE QUANTUM WAVE-VORTEX: INTERPRETATION a≈1/137 AS A WAVE CONSTANT

George Kirakosyan
Physics Department, State Engineering University
Yerevan, Armenia
P.O. Box 42, Dubai, UAE
gevorge@inbox.ru

Abstract

The connection of alpha ($\alpha \approx 1/137$) to redistribution of intensities in interference of circularly polarized waves it has shown. Obtained number coincides to known one in reached accuracy: 10^{-10}. The photon represented as a quantum wave packet. The electron's model proposed as Compton's circularly polarized standing wave. The origins of the mass and static fields (charges) interpreted as a relativistic mass and pseudo static electromagnetic fields ("halos") arising in interference of quanta. Electron's magnetic moment and g value obtained with 10^{-10} accuracy. Physical interpretation of de Broglie's wave is proposed.

Keywords: Elementary Particle; Elementary Charge; De Broglie Wave; Compton's Wavelength; Fine Structure Constant; Coupling Constant.

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Introduction

As known, there are no theoretical or conceptual accomplished interpretations in contemporary physics of the nature of the basic particle electron as well as of the existence of a fine structure constant $\alpha \approx 1/137$ having great importance in microcosm. Its may be obtained from experimental measurements only according to standard formalism. Many of renowned physicists (such as P. Dirac and R. Feynman) have attempted to obtain α theoretically, which continues to be an open question. We have looked at the problem of fine structure constant in conjunction to the global problem of revealing the physical nature of elementary particles, since α appears indivisible from them, as their deeply peculiarity. It is possible to judge the extreme importance and all complications related to this dimensionless constant from [1]. The continuous attempts to present α by means of artificial combinations of other known constants (numerological representations, etc.) not considered as theoretical interpretations. We will refer to Feynman's known critical remark [2] on this issue. The long-term unsuccessful efforts to obtain α theoretically force us to refer to wave-particle duality principle applied in quantum representations. By mentioning the large circle of phenomena in microcosm where α exposes as an important parameter, we bring also some expressions below related to description of Hydrogen's atom that help us to realize direct interconnection of α with elementary particles (photon, electron). Using known relations $e=(2\varepsilon_0\alpha hc)^{0.5}$ and $m_e=h/c\lambda_e$ we express the speed of electron v_0 on the first Bohr's orbit, the orbit's radius a_0 and Rydberg's constant R by simplest expressions, containing only α, c and Compton's wave length of the electron λ_e:

$$v_0 = \alpha c\,, \quad a_0 = \lambda_e / 2\pi\alpha \approx 0{,}53 \cdot 10^{-10} m\,, \quad R = \alpha^2 c / 2\lambda_e \approx 3{,}3 \cdot 10^{15} s^{-1}$$

From these expressions, we look at α as an independent universal numeric constant defining the **dynamical, geometrical** and **wave** properties of localized particles as well as of the non-localized particles (photons). Mentioned view is pointing to the existence of a certain general principle in formation of all kinds of elementary particles and, to the possibility of linking α to unique nature of localized and non-localized quantum objects.

Our attempts to interpret fine structure constant as well as basic particles correspond to wave-field principle of primordial substance. Einstein, Schrodinger, Heisenberg and other classics of past century were convinced supporters of such approach. We can remark [3], [4] as recent works pointing on this direction. We attempt to show that de Broglie's wave-particle duality principle, electrodynamics and special relativity (STR) allow representing

photon as well as the localized particle (electron) from wave-field point of view.

1. Deduction of $a \approx 1/137$ as a wave interference redistribution constant

In this chapter we prove the equation:

$$\sum I_m / I \approx 0.085424 \approx e_* = \alpha^{0.5} \qquad (1)$$

Where: I_m is the intensity of m peak. I is the total intensity of the circularly polarized interfering waves, $\alpha \approx 1/137$ is the Fine Structure Constant, e_* is the value of the elementary charge in the natural system of units; $c = \hbar = 1$. To prove (1) we represent the interference as a standing wave appearing in Compton's localized circularly polarized waves (Fig.1)

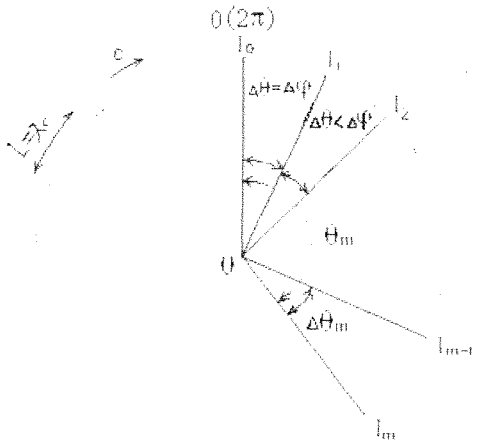

Figure 1. Angular distribution of intensities in interference of circularly polarized waves

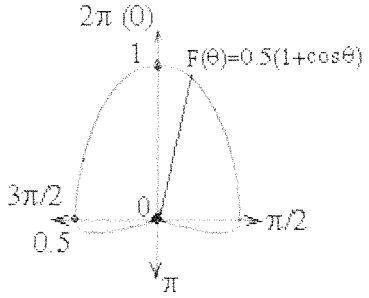

Figure 2. The Kirchhoff's function

We have chosen described model of the wave interference as a classical analog to the standing de Broglie's wave on the first Bohr's orbit, implementing following replacements: $l_{orb} = \lambda_c$, where λ_c is the Compton's wavelength and $v_{orb} = c$. We consider number n of interfering waves as much greater than one, which corresponds to existing classical representations of quanta. We have used handbook equations (2), (3) to describe the relations

between amplitudes and intensities [5] supposing that examined interference satisfies to Huygens – Fresnel's principle.

$$\frac{A_m}{A_0} \approx \frac{2}{(2m+1)\pi}, \quad \frac{I_m}{I_0} = \frac{A_m^2}{A_0^2} \approx \frac{4}{(2m+1)^2\pi^2} \tag{2}$$

Where: A_m is the amplitude of m peak. A_0 is amplitude of θ – peak (main), m = 1, 2, 3 ... n. Since the equations (2) are approximations suitable for small angular distribution only, for their implementations to infinite angular distribution we have used the Kirchhoff's function (3), considering amplitudes dependence on direction, according to Huygens – Fresnel's principle:

$$F(\theta) = 0{,}5(1 + \cos\theta) \tag{3}$$

Where: equation (3) satisfies conditions, $F(\theta) = 1$ at $\theta = 0$ (maximum of amplitude on direction "forward") and, $F(\theta) = 0$ at $\theta = \pi$ (the amplitude becomes zero on direction "backward"), (Fig. 2). Using equation (3) from equation (2) we obtain:

$$\frac{A_m}{A_0} \approx \frac{2F(\theta)}{(2m+1)\pi} = \frac{1+\cos\theta_m}{(2m+1)\pi} \tag{4}$$

According to initial conditions (Fig.1) the angular distance between first and main peaks will be equal to average value of a phase difference $\Delta\varphi$ for the interfering waves. The angular distances between two consecutive peaks $\Delta\theta_m$ will be consequently decreasing as described further. Considering that, amplitudes of the secondary peaks differ from each other by phase $(\pm 2\pi n)$ we can directly summarize:

$$\sum I_m = I_1 + I_2 + \ldots + I_n = A_1^2 + A_2^2 + \ldots + A_n^2 \tag{5}$$

From equations (4) and (5) follows:

$$\frac{\sum I_m}{I_0} \approx \frac{1}{\pi^2} \sum_{m=1}^{n} \left(\frac{1 + \cos\theta_m}{2m+1} \right)^2 \tag{6}$$

The secondary peaks are differs at main peak by phase; $\pi/2 \pm 2\pi n$ because the main peak corresponds to 0 by $\Delta\varphi$, meanwhile secondary peaks correspond to; 3π, 5π... $\pi(2m+1)$. Considering the above, we define the distribution of total intensity as:

$$I^2 = I_0^2 + (\sum I_m)^2$$

From above follows:

$$\sum I_m / I_0 = \tan\Delta\varphi, \quad \sum I_m / I = \sin\Delta\varphi, \quad I_0 / I = \cos\Delta\varphi \tag{7}$$

Using (7) in (6) we obtain:

$$\frac{1}{\pi^2}\sum_{m=1}^{n\to\infty}\left(\frac{1+\cos\theta_m}{2m+1}\right)^2-\tan\Delta\varphi\approx 0 \qquad (8)$$

To find functional link between θ_m and $\Delta\varphi$ we use vector diagram (Fig. 3).

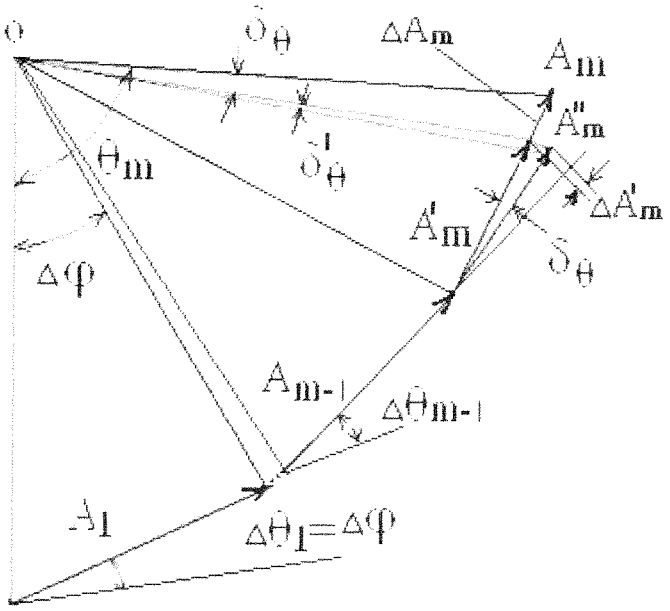

Figure 3. Vector representation of secondary interferential maximums

$A_1 \ldots A_m$	Vectors of secondary maximums corresponding to initial equations: [see eq. (2)]
A_m^I	The vector of m maximum after first correction: [see eq. (10)]
A_m^{II}	The vector of m maximum after second correction: [see eq. (12)]
θ_m	The angular shift of m maximum at the main; [see (Fig. 1)]
$\Delta\varphi$	The phase shift of interfering waves
ΔA_m, δ_θ	The first corrections corresponding to equation (10)
ΔA^I_m, δ^I_θ	The second corrections corresponding to equations (11)

Table 1. Explanatory to Fig. 3.

With application of equation (4) instead of (2) small changes of vectors of interfering waves arise as a function at θ_m. Aftermath of that the secondary peaks also will change, by values as well as by locations. The angular distances between two peaks will changed as illustrated in diagram. Some reduction of angle θ_m occurs because of reduction of the vector A_m aftermath of replacement (2) with (4). The correction for the angle $\Delta\theta_m$ we define as:

$$\delta_\theta \approx \frac{\Delta A_m}{A_0}\Delta\varphi \approx \left|\frac{2}{(2m+1)\pi} - \frac{1+\cos\Delta\varphi m}{(2m+1)\pi}\right|\Delta\varphi \approx \frac{\Delta\varphi(1-\cos\Delta\varphi m)}{\pi(2m+1)} \qquad (9)$$

Using equation (9) and considering the relative change of angle θ_m the equation (4) becomes:

$$\frac{A^I{}_m}{A_0} \approx \frac{1+\cos\ \Delta\varphi m(1-\delta_\theta/\Delta\varphi m)}{(2m+1)\pi} \qquad (10)$$

Simultaneously, with reduction of angle between directions A_m and A_{m-1}, the vector $A^I{}_m$ will slightly turn to right, as a result it becomes $A^{II}{}_m$. For this reason the projection of $A^{II}{}_m$ on a direction A_{m-1} increases, that leads to relative increase of their sum by value: $1+(\delta_\theta/\Delta\varphi m)^2$. Mentioned factor leads to a new small change of the angle and causes a new small increase in the vectors sum. We can continue these reasoning infinitely which brings to amendments in the form of Maclaurin series, for the angles and for the vectors accordingly:

$$(\Delta\varphi m)^1 = \Delta\varphi n - \delta_\theta[1+(\delta_\theta/\Delta\varphi n)+(\delta_\theta/\Delta\varphi n)^2+\cdots+(\delta_\theta/\Delta\varphi n)^n] = \Delta\varphi n - \delta_\theta/(1-\delta_\theta/\Delta\varphi n)$$

$$A_m^{11}/A_m^1 = 1+(\delta_\theta/\Delta\varphi n)^2+(\delta_\theta/\Delta\varphi n)^4+\cdots+(\delta_\theta/\Delta\varphi n)^{2n} = 1/\left[1-(\delta_\theta/\Delta\varphi n)^2\right] \quad (11)$$

Considering relations (11) we have replaced θ_m in (8) resulting to below equation:

$$\sum_{m=1}^{n\to\infty}\left(\left[\frac{1+\cos[\Delta\varphi n-\delta_\theta/(1-\delta_\theta/\Delta\varphi n)]}{(2m+1)}\right]\times\left[\frac{1}{1-(\delta_\theta/\Delta\varphi n)^2}\right]\right)^2 \approx \pi^2\tan\Delta\varphi \quad (12)$$

Where: δ_θ as defined above [see equation (9)]

By method of insertion, using numeric calculating, the value satisfying to equation (12) has found:

$$\Delta\varphi \approx 0.0855287810\,2 \quad (13)$$

According to equations (7) using result (13) we obtain:

$$\sum I_m / I = \sin\Delta\varphi \approx 0.0854245428\,6 \approx e_* \quad (14)$$

Thus, the result (14) confirms initial assumption and equation (1).

Obtained number coincides with elementary charge in relative units in achieved accuracy range of measurements; it corresponds to value of Fine Structure Constant:

$$1/\sin^2\Delta\varphi \approx e_*^{-2} \approx 137.0359999 \approx 1/a \quad (15)$$

Here are results of last precious measurements *1/a*:

$$1/a \approx 137.0359990 \div 137.0359998 \quad [6], [7]$$

Thus, as shown, there exists a constant, referring to interference of circularly polarized waves in generalized condition, which correlates to the basic coupling constant. We will interpret *a* as well as the "Interferential Redistribution Constant" considering above.

2. Description of the photon as a quantized wave packet

2.1. We start from classical wave equation:

$$\Delta s = \frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 s}{\partial y^2}+\frac{\partial^2 s}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 s}{\partial t^2}$$, where: $\Delta = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is the

Laplace's operator, *s* characterizes the amplitude of perturbation and *v* is propagation speed.

For the harmonic oscillations and sinusoidal waves takes place:

$$\frac{\partial^2 s}{\partial t^2} = -\omega^2 s \quad (16)$$

Where: ω is the cyclic frequency of the wave

For the vacuum $\varepsilon=\mu=1$ and Maxwell's equations in vector form become:

$$rot\mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t},\ rot\mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t},\ div\mathbf{E} = 0,\ div\mathbf{H} = 0 \qquad (17)$$

From (17) follows:

$$\Delta\mathbf{E} = \varepsilon_0\mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2},\ \Delta\mathbf{H} = \varepsilon_0\mu_0 \frac{\partial^2 \mathbf{H}}{\partial t^2}$$

Considering; $\varepsilon_0\,\mu_0=1/c^2$ we write:

$$\Delta\mathbf{E} = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2},\ \Delta\mathbf{H} = \frac{1}{c^2}\frac{\partial^2 \mathbf{H}}{\partial t^2} \qquad (18)$$

Where the equations (18) satisfy to equation (16)

Considering: the rotor form of Maxwell's equations, that free oscillation may only be harmonic, two mutually perpendicular oscillations of **E**, **H** vectors in a wave flow are equivalent in all means (that looks better from equations (18)), we examine below equations as a particular solution of Maxwell's equations, satisfying mentioned conditions, within conformity to graphic image (Fig. 4)

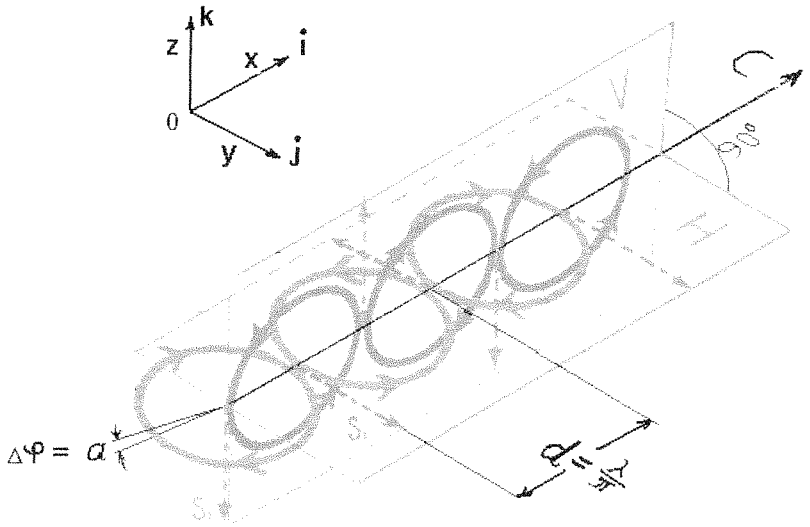

Figure 4. Schematic representation of the photon's wave flow

$$\mathbf{E} = E_0[\mathbf{i}\sin(\omega t + \alpha) + \mathbf{k}\cos(\omega t + \alpha)]$$
$$\mathbf{H} = H_0[\mathbf{i}\sin(\omega t + \beta) + \mathbf{j}\cos(\omega t + \beta)] \qquad (19)$$

Equations (19) correspond to circularly polarized, mutually perpendicular oscillations that correspond to circle's equation in geometric meaning.

Where: E_0, H_0 are the modules of amplitudes of field's tension, $\omega = 2\pi c/\lambda$ is the cyclic frequency of oscillation, λ wavelength, α, β phases of oscillations, **i, j, k** the unit vectors in cartesian coordinate frame (Fig. 4)

The equations (19) are strongly right for an infinite wave flow only; that is the sinusoidal wave. Supposing the ideality of sinusoidal wave flow and the absolute stability of its parameters, considering the constant linear speed of field's circulation (as example; from equations (19) it follows: $V_H = (V_x^2+V_y^2)^{0.5} = c$. Where: V_H is the linear speed of circulation in the "horizontal rings", $V_x=c\sin\omega t$, $V_y = c\cos\omega t$) we got the field's circulation by the circle with λ length. Thus, this ideal imagination brings to infinity of energy's density: $\rho\rightarrow\infty$. Considering infiniteness of the sinusoidal wave flow also, we cannot judge anything certainly about total energy of described flow. The similar serious difficulties have risen in early attempts to interpret the mass of localized particles within electromagnetic origin. We attempt to represent the photon as a wave flow having restricted length, which is not sinusoidal. Considering established properties of photon, we will define the length of wave flow as:

$$L \approx \lambda n \approx c\tau \qquad (20)$$

Where: n is average quantity of whole waves composing the flow in its stable condition, τ is action time of photon (time of its radiation or, absorption). For example; for the visible light L is about *meter*, λ is about *micron* and;

$$n \approx L/\lambda \approx 10^6 >> 1$$

Considering (20) we can look at photon's wave flow as a "part of sinusoid" (or, as a "wave packet" as per accepted terminology) within approximations.

2.2. Considering that Maxwell's equations already satisfy to Lorentz transformations and STR (because the propagation speed of electromagnetic wave; $v = const = c$ in all inertial systems), from above mentioned conditions we can judge that our model will be not much far from reality. We can make certain conclusions on this base. Realizing that our judgments and results have approximate meaning, we write the equations of the neighboring pair of whole waves in a restricted wave flow resulting from (19) as:

$$\mathbf{E}_V \approx E_0[\mathbf{i}\sin(\omega t + a) + \mathbf{k}\cos(\omega t + a)]$$
$$\mathbf{E}_H \approx E_0[\mathbf{i}\sin(\omega t + \beta) + \mathbf{j}\cos(\omega t + \beta)] \qquad (21)$$

Here we use the same symbol **E** with different indexes to emphasize the symmetry and full equality of two mutually perpendicular circulations in a flow. We do not examine the initial phases of oscillations, which now are out of our study. According to equations (21) the concentration of energy of single circulation has to be of "linear" character [1] in the form of "string-ring". We will imagine the energy's distribution as a torus, the section's diameter of which is small compared to its length. We define the energy of one "ring" within classical representation as field's energy:

$$\varepsilon_1 \approx \rho V \approx E_0^2 V \tag{22}$$

Where: E_0 is the amplitude of field's tension, $V \approx \sigma\lambda$ is the volume of its concentration, σ is the section of the "ring"

For total energy in a flow, we write:

$$w \approx 2nE_0^2\sigma\lambda \tag{23}$$

Where: n is the number of pairs of "rings" that we consider as the number of whole waves.

The wave flow will show some deviation in parameters as a "part of sinusoid". Particularly, the number of whole waves will be an average: $n \approx n_0 \pm 1/2$. To interpret aforesaid we can image propagation of wave flow as a permanent process of originating new "rings" ahead, with simultaneously annihilation of "rings" end of the flow. In conformity to this imagination, the instant number of whole waves will be changeable in range:

$$\Delta n \approx \pm 1/2 \text{ or, relatively: } \Delta n / n \approx 1/2n \tag{24}$$

The coherence's time considered as the average time of photon's action in quantum representations (see "quantum optics"):

$$\tau \approx \tau_c \approx \pi / \Delta\omega = 1/2\Delta\nu \tag{25}$$

Where: ν is the photon's frequency; $\Delta\nu$ is the Heisenberg's uncertainty for frequency.

From equations (20), (25) we get: $n \approx c\,\tau/\lambda = \nu\,\tau \approx \nu / 2\Delta\nu$. From equation (24) we see:

$$\Delta\nu / \nu \approx 1/2n \approx \Delta n / n \tag{26}$$

We make first important conclusion from above coincidence:

a). *The examined model of photon shows the equality of Heisenberg's Uncertainties with the deviation of parameters in a restricted wave flow* (the

---

[1] Assuming other characters of energy's distribution in wave flow ("volume" or, "surface" character) we get other appraises for L, τ, which become unconformable to actual exposed ones.

phenomenon known as the "beating of wave"). This conclusion shows the compatibility of Heisenberg's Uncertainties (quantum concept) with the deviations of parameters (wave concept) in a restricted wave flow.

However, the seeming main contradiction between wave and quantum representations is the total difference in expressions of energy, accordingly: $\varepsilon \propto A^2$ and, $\varepsilon \propto \nu$. Where: A is the amplitude; ν is the frequency of the wave. Attempting to solve this contradiction, we will use conclusion a). First, we define the uncertainty for the single whole wave's energy as:

$$(\Delta\varepsilon_1 / \varepsilon) = (\Delta\nu / \nu) / n = (\Delta\lambda / \lambda) / n \approx 1 / 2n^2 \qquad (27)$$

We relate the transformation of energy between neighboring whole waves (coupling energy) in propagation process to a and to uncertainty by equation (27) as:

$$\Delta\varepsilon_{e\mu} / \varepsilon \approx \alpha(\Delta\varepsilon_1 / \varepsilon) \approx \alpha / 2n^2 \qquad (28)$$

Assuming $\Delta\varepsilon_{e\mu}$ as a possible minimal portion of energy (quanta of energy) for the examined wave flow we define it as:

$$\Delta\varepsilon_{e\mu} = h \cdot \mathrm{s}^{-1} \qquad (29)$$

Considering equation (28) we write:

$$\varepsilon \approx 2n^2 \Delta\varepsilon_{e\mu} / a \approx 2n^2 h / a \cdot \mathrm{s} \qquad (30)$$

Equalizing this value to actual energy of photon, we define its frequency:

$$\varepsilon = h\nu \approx 2n^2 h / a \cdot \mathrm{s}$$

$$\nu \approx 2n^2 / a \cdot \mathrm{s} \qquad (31)$$

To define the number of whole waves in the flow we considering the section size of "ring-string" conditioned by changeability (uncertainty) of wavelength as:

$$\sigma \approx (\Delta\lambda)^2 \qquad (32)$$

We define the spatial uncertainty of distribution for the energy as:

$$\Delta V / V \approx \sigma\Delta\lambda / \lambda^3 \approx (\Delta\lambda / \lambda)^3 \qquad (33)$$

Considering universality of α (chap 1), we assume:

$$\Delta\lambda / \lambda \approx \eta\alpha \qquad (34)$$

Where: η is a coefficient, which will be discussed.

According to (34) we interpret α as well as "natural uncertainty of quanta" that equal to the "wave interferential redistribution constant" (chapter 1).

We consider below equation as a condition of stability of wave flow:

$$1 / n \approx (\Delta V / V) \approx (\eta\alpha)^3 \qquad (35)$$

We call equation (35) condition of "Symmetry of Uncertainties' Distribution" (SUD)

2.3. To test our formulas, for $\eta=1$ we get from equation (35); $n_1 \approx 1/\alpha^3 \approx 2.6\cdot10^6$ and from equation (31) we define; $\nu \approx 1.8\cdot10^{15}$. Using equation (20) we get: $\tau \approx \lambda n/c = n/\nu \approx 1.4\cdot10^{-9}$ s.

This numbers are conformable with Rydberg's constant and to known handbooks appraisements for atomic photons. To test these expressions for other energy level we have used experimentally established properties for γ quanta; $\varepsilon \approx 0.5$ *Mev* ($\nu \approx 10^{20}$ s^{-1}), $\tau \approx 10^{-12}$ s. Using equation (31) we obtain; $n \approx (\alpha\nu\cdot s/2)^{0.5} \approx (10^{20}/2\cdot137)^{0.5} \approx 6\cdot10^{8}$. From equation (20), we define $\tau \approx n/\nu \approx 6\cdot10^{-12}$s that coincides with the actual one.

These two examples show the "workability" of examined model in a large interval of energy.

2.4. Equalizing equation (23) with the actual energy of photon; $h\nu$ we write:

$$2E_0^2\sigma\lambda n = h\nu \tag{36}$$

The left and right sides of equation (36) are the total energy of wave flow, within classical and within quantum representations accordingly.

From equation (36) we define; $E_0 = (\omega/2\pi)(h/2c\sigma n)^{0.5}$. According to Fig. 6, we can define the phase shift $(1/\omega)$ for $\mathbf{E}_V$, $\mathbf{E}_H$ and write equations (21) as:

$$\mathbf{E}_V \approx \frac{\omega}{2\pi}\sqrt{\frac{h}{2c\sigma n}}\left[\mathbf{i}\sin\omega(t-\frac{x}{c}) + \mathbf{k}\cos\omega(t-\frac{x}{c})\right]$$

$$\mathbf{E}_H \approx \frac{\omega}{2\pi}\sqrt{\frac{h}{2c\sigma n}}\left[\mathbf{i}\sin\omega(t-\frac{x}{c}-\frac{1}{\omega}) + \mathbf{j}\cos\omega(t-\frac{x}{c}-\frac{1}{\omega})\right] \tag{37}$$

Resulting from above, we mark:

b). *The equations* (37) *are approximate form* [2] *of quantized wave equations, which simultaneously satisfy to classical conceptions taken for the single whole wave, as well as to quantum conceptions taken for the restricted wave flow.*

---

[2] To find the exact form of these equations it's necessary to establish the correct functions; $\omega = F_1(n)$, and $\sigma = F_2(n)$ that we have done approximately. We can expect that these functions will contain specific factors similar to Fourier or, Maclaurin series arising because restrictions of photons wave flow. This issue can be subject to study the theoretically as well as experimentally.

As shown above, mentioned consensus becomes possible in case of "String-ring" form of energy's concentration in the wave flow.

2.5. The presented approximate representation provides an opportunity to move ahead and make new conclusions.

Attributing to photon an impulse; $P = \hbar k = \hbar\omega/c$ and the relativistic mass $m = \hbar\omega/c^2$ in conformity to quantum representation and STR, we image distribution of mass by the length of "rings". From graphic image (Fig. 4) and equation (36) we define the moment of impulse for each "ring" as:

$$S_1 = cm_1 r = cmr/2n = c(\hbar\omega/c^2)(\lambda/2\pi)(1/2n) = \hbar/2n$$

The scalar sum of moments in the wave flow we define as:

$$S = \sum S_1 = 2nS_1 = \hbar \qquad (38)$$

Considering spatial distribution and directions of circulations (Fig. 4) for the vector sum of moments we get:

$$\mathbf{S} = \sum \mathbf{S}_1 = 0 \qquad (39)$$

From equations (38), (39) we make conclusion:

c). *The summary impulse moment (scalar sum) of the photon is equal to* $\hbar$ (accepted as unit).

d). *The photon's spin distributed discretely, by wave flow's length within mutually perpendicular directions, perpendicular to direction of its propagation* (Fig. 4)

3. Description of the electron as a Compton's standing wave

3.1. We start with examination of the meaning of η in SUD (*2.2.*). We have accepted in above example (*2. 3.*) $\eta = 1$. It means $n = 1/\alpha^3$ number of whole waves filling a full length of wave flow. From examined model of photon becomes clear that n is an individual parameter of quanta defining its peculiarities. We are free to suppose that on the length of a flow can be located $n = (2/\alpha)^3$, $(3/\alpha)^3$...etc, whole waves, as well as we can take; $n = (1/2\alpha)^3$, $(1/4\alpha)^3$...etc, which means we assume; $\eta = m/p$, where m, p are whole positive numbers. In such way, we can "construct" photons having different energies and parameters, satisfying (approximately) to actually exposed properties of photons. We examine now the special condition: $\eta \approx 1/2\pi$. We suppose that in this case the quant is able to form localized condition. We imagine above said as a possibility to "wrap" a restricted wave flow symmetrically by volume. We consider the mentioned condition necessary for stability of the localized quant. The graphic illustration of described concept presented (Fig. 5).

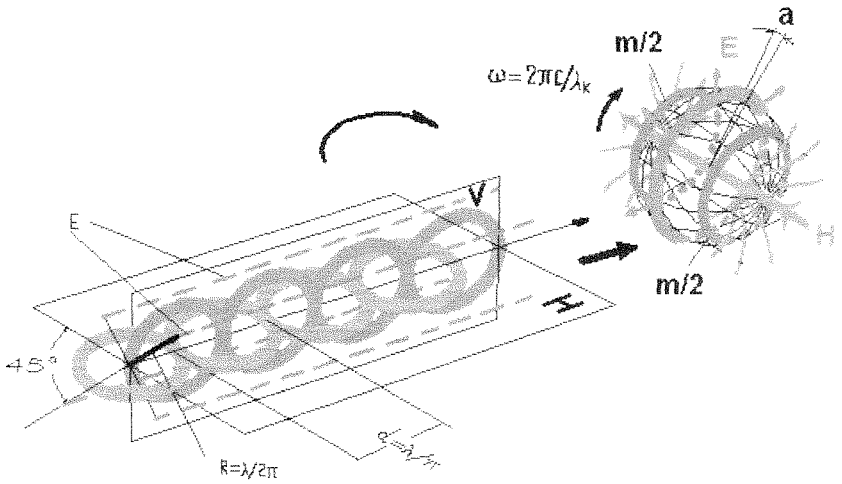

Figure 5. The localization of quant

Supposing that wave flow "wraps" in vertical flat by *0* radius, we conclude that "vertical rings" simply become to the same place, meanwhile the "horizontal rings" are distributed in space within axial symmetry. We "construct" the elementary particle as the localized wave-vortex (Fig. 6) by means of described mental operation [3].

3.2. We define the energy *(v)* corresponding to value: $\eta \approx 1/2\pi$ from equations (31), (35):

$$n \approx (1/\eta\alpha)^3 = (2\pi/\alpha)^3 \approx 6.4 \cdot 10^8 \qquad (40)$$

$$\nu \approx 2n^2/\alpha \cdot s \approx 1.12 \cdot 10^{20} s^{-1} \qquad (41)$$

Obtained number (41) definitely is near to electron's rest energy ($\nu_e \approx 1.24 \cdot 10^{20}$ s^{-1}). Considering the mentioned coincidence we examine described localized quant as the electron's model. We present rest mass of the particle as a relativistic mass of quant:

$$m_e = \varepsilon / c^2 = h\nu_e / c^2 = h / c\lambda_e \qquad (42)$$

Where: $\lambda_e \approx 2.426 \cdot 10^{-12}$m is the Compton's wavelength for the electron.

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[3] Conservation's laws prohibit the transformation of single photon to a single localized particle.

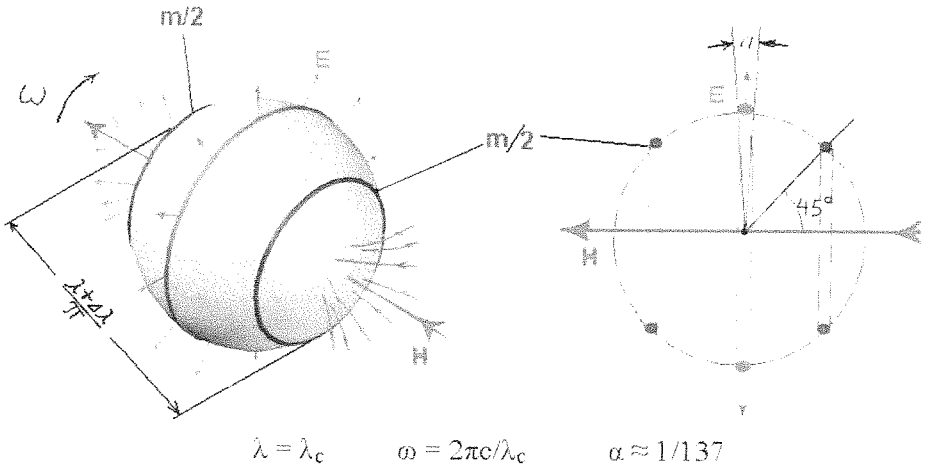

Figure 6. The model of the electron

Considering previous interpretations of a (chapters 1, 2) we represent m_e as the energy of main interferential maximum in localized quanta. We represent the electromagnetic energy of electron as the energy of pseudo static fields, which are conditioned by energies of secondary interferential maximums (we can imagine it as "halos" of interference). Within conformity to above said we define:

$$\varepsilon_{e\mu} = \varepsilon_e + \varepsilon_\mu = \alpha h\nu = \alpha hc / \lambda_e = \alpha c^2 m_e \qquad (43)$$

Where: ε_e, ε_μ are energies of electric and magnetic fields accordingly. Their equality follows from initial condition (21). From equation (43), we write:

$$\varepsilon_e = \varepsilon_\mu = 0.5\varepsilon_{e\mu} = 0.5ah\nu = \alpha hc / 2\lambda_e \qquad (44)$$

The quantity and locations of "rings" for masses as well as for secondary maximums presented in graphic (Fig. 6). We come to presented image considering previous conclusions and certain reasons about symmetry of "construction" of formed particle.

We present the electrical energy (44) as traditional "charged" sphere's energy:

$$\varepsilon_e = q^2 / 8\pi\varepsilon_0 r \qquad (45)$$

Where: ε_0 is the electric constant. Equalizing equations (45) and (44), considering $r = \lambda_e/2\pi$ we get:

$$q = \sqrt{2\varepsilon_0 \alpha hc} = \pm e \qquad (46)$$

3.3. We define particle's moment impulse (spin) by analogy to photon's spin (*2.5.*), (Fig. 6). Considering: $r = \lambda_e/2\pi$ and (42) we get:

$$S = 2(\frac{1}{2} m_e r \sin\frac{\pi}{4} c \sin\frac{\pi}{4}) = \frac{m_e rc}{2} = \frac{\hbar}{2} \qquad (47)$$

3.4. To define particle's magnetic moment, considering equation (44) we present it as produced from circulation of the "elementary charge" by its diametric length:

$$\mu = ecr/2 = ec\lambda_e/4\pi = ech/4\pi\, cm_e = e\hbar/2m_e = \mu_B \qquad (48)$$

Where: $\mu_B = e\hbar/2m_e$ accepted as unit (*Bohr magneton*). The results (46), (47), (48) are in conformity with actual values of charge, spin and magnetic moment of the electron (see: *electron's g factor*).

We can make correction to μ considering previous representations. Some enlargement in the sizes of the particle will occur as localized wave, in view of its uncertainty. That brings to a corresponding small change of magnetic moment. We will define the actual magnetic moment of electron using expression:

$$\mu_e \approx \mu\,(1 + k_1 + k_2 + k_3 + ... + k_n) \qquad (49)$$

We define correction factor k_1 as a parameter of enlargement of particle's diameter caused by natural uncertainty of quanta (34), (Heisenberg's uncertainty for the localized quantum):

$$k_1 = \Delta r/r = \Delta\lambda/\lambda = \alpha/2\pi \approx 0.001161409725.. \qquad (50)$$

We can interpret k_1 as an illustration to *Swinger's Correction.*

We define k_2 as a factor of smallest reduction of effective radius R_{ef} of circulation in relation to H axis, caused by enlargement of "charge's" distribution, corresponding to angle α within conformity to previous point (see Fig.7)

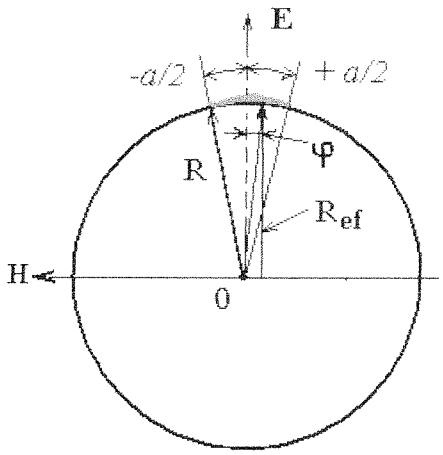

Fig. 7. The distribution of electron's charge (*a* shown much bigger)

We obtain from figure:

$$k_2 = \frac{\Delta R}{R} = \frac{R_{ef} - R}{R} = \frac{1}{\alpha}\int_{-\alpha/2}^{+\alpha/2} \cos\varphi \, d\varphi - 1 = \frac{2\sin\alpha/2}{\alpha} - 1 \approx -2.21888 \cdot 10^{-6} \qquad (51)$$

Thus, the factors (k_1, k_2) obviously derives from examined model. Their sum gives:

$$\mu_e / \mu_B \approx 1 + k_1 + k_2 \approx 1.0011591908 \ldots \qquad (52)$$

This number differs from experimentally measured one by 10^{-8} digit only. To define k_3 we assume that it conditioned by non-homogeneity of "charge's" distribution in the range of angle α. We test this assumption in the form of an excitation (by analogy of QED methods) considering equations (35), (40) within conformity to below expression:

$$k_3 = \alpha^3 (1 + 1/2\pi + 1/4\pi^2 + \cdots + 1/2^n \pi^n) \qquad (53)$$

Using Maclaurin's series formula, we get:

$$k_3 = a^3 / (1 - 1/2\pi) \approx 4.621146 \times 10^{-7} \qquad (54)$$

Considering k_3 we get:

$$\mu_e / \mu_B \approx 1 + k_1 + k_2 + k_3 \approx 1.0011596529 \ldots \qquad (55)$$

This number differs from the measured one by less than 10^{-10} digit:

(1.0011596522…) [6]

New small corrections are possible define as effects of mutual actions of mentioned factors. However, we cannot be sure in rightness of ours results because (55) already is comparable to experimental capabilities.

3.5. In conformity to photon's model, (chap. 2) there is no difference between mutually perpendicular two vectors $\mathbf{E_v}$, $\mathbf{E_h}$. It means the "left" and "right" systems, formed by two vectors with the vector of propagation, will be compatible. Thus, the photon's "mirror particle" will be the same as the original, which means it cannot have its "antiparticle". The mentioned equality of two vectors becomes disturbed in localized quantum (Fig. 5, 6) because different kind of symmetry (axial and central) arises for its pseudo static fields (in distributions of electric and magnetic "charges"). It means the localized quanta and its "mirror particle" become incompatible. This illustration allows interpreting the existence of particle-antiparticle pairs for localized quanta as its "left" and "right" circulations, as well as contrary signs of "charges" (*3.2.*)

3.6. To test the "workability's" of electron's model we examine the possibility of communicating it with de Broglie's wave. De Broglie's wave presented in handbooks as:

$$\Psi(\mathbf{r},t) = Ce^{i(\frac{Et}{\hbar} - \frac{\mathbf{pr}}{\hbar})} \tag{56}$$

Where: $\mathbf{r}$ is radius vector of free point, t is time, E is energy of moving particle, $\mathbf{p}$ impulse.

It has shown in courses that propagation speed of de Broglie's wave coincides to particle's speed within all directions:

$$V_x = \partial E / \partial p_x,\ V_y = \partial E / \partial p_y,\ V_z = \partial E / \partial p_z$$

$$\text{Or, in vector form: } \mathbf{V} = \nabla_{\mathbf{p}} E = \mathbf{v} \tag{57}$$

The equation (57) means that de Broglie's wave moves with the particle. Its wavelength connected to the particle's impulse as $\boldsymbol{p} = \hbar \boldsymbol{k}$ where: $k = 2\pi/\lambda$. For the low speed; v << c the energy becomes; $E = p^2/2m_0$ and wave length becomes:

$$\lambda_D = \frac{2\pi\hbar}{\sqrt{2Em_0}} \tag{58}$$

We will replace λ_D by corresponding frequency and write equation (58) as:

$$\nu_D = \frac{c}{\lambda_D} = \frac{c\sqrt{2Em_0}}{2\pi\hbar} \tag{59}$$

We represent a mass of particle as relativistic energy of localized quanta as per examined model: $m_0 = h\nu_c/c^2$ (*3.2.*). Expressing kinetic energy as $E = m_0V^2/2$, from equation (59) we get:

$$\nu_D = \nu_c(\mathrm{v}/c)\ .\ \text{Or, for the wavelength; } \lambda_D = \lambda_c(c/V) = h/m_0V \qquad (60)$$

Where: λ_c, ν_c are Compton's wavelength of particle and its frequency (as a localized quanta).

The equations (60) correspond to Doppler's known effect by its form, which points on the physical meaning of de Broglie's wave.

3.7. We will show below that it is possible to get the same result and conclusion from equations (37) without referring to quantum representation. For simplicity we will examine the movement of one "vertical ring" only (Fig. 4), assuming the observation point is *0* and movement directed by *x* axis (Fig. 8).

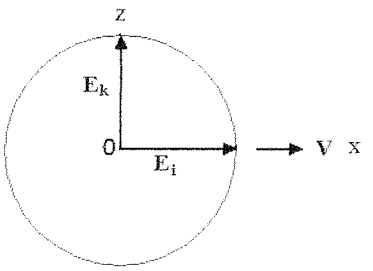

Fig. 8. The movement of a localized quant's

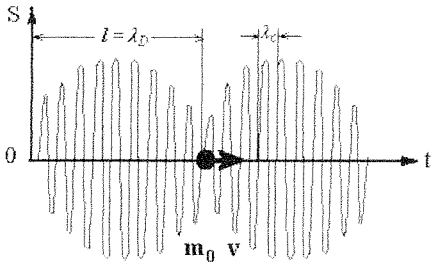

Fig. 9. De Broglie's wave illustration

The oscillations $\mathbf{E_k}$, $\mathbf{E_i}$ will look in observer's system with some changed frequencies as transverse and longitudinal Doppler effects accordingly. For frequencies we write:

$$\omega_k = \omega_c\sqrt{1-(\mathrm{v}/c)^2}\ ,\ \omega_i = \omega_c\sqrt{\frac{1-\mathrm{v}/c}{1+\mathrm{v}/c}}$$

Accepting; v << *c* we write: $\omega_k \approx \omega_c$ and $\omega_i \approx \omega_c(1-\mathrm{v}/c)$. We define summary oscillation as per sum of two oscillations having close frequencies (see handbook):

$$S = 2A_0\cos(\frac{\omega_k-\omega_i}{2}t)\sin(\frac{\omega_k+\omega_i}{2}t)$$

Considering above we get:

$$S = 2A_0 \cos(\frac{v\omega_c t}{2c}) \sin \omega_c t \qquad (61)$$

Equation (61) corresponds to "wave beating" as shown in graphic (Fig. 9). Accepting; $\omega_c = \omega_e$ we define the length of one "beating packet" as:

$$l = (1/2)(2c/v)(2\pi\ c/\omega_e) = \lambda_e(c/v) = h/m_0 v = \lambda_D \qquad (62)$$

Presented interpretation clarifies the physical meaning of de Broglie's wave as *Doppler Effect arising from movement of Compton's standing wave* (elementary particle).

3.8. Comparing the examined model of the electron with experimentally established its peculiarities we can see some opportunities that can help us to interpret these. Particularly, the intriguing problem of defining the actual size of electron gets new aspects. As it seems from graphic, (Fig. 6) the electron is mostly "empty" by its "construction" (similar to atom's "construction"). This circumstance may open new possibility to explain its interaction with the high-energy hadrons and other heavy particles (for example, it seems probable that heavy particles can just pass through the electron without seeming energetic transformations). It explains the seeming "absence of sizes" of the electron, although its actual size is much bigger than hadrons. On the issue we can point on theoretical conclusions of some renowned researchers about; *"Impossibility of localization of the electron in a space less than Compton's wavelength"* (L. Landau, R. Peirls).

We remark that examined model removes some serious problems as well, concerning to "infiniteness" of electron's electrical energy, to its spin and "rotation speed" etc, which arise from its representation as "material point".

We remark [8] as a resent conclusion pointing to "string-ring" form of the electron's mass by size comparable to Compton's wavelength. The mentioned aspects maybe subjects to future study.

Discussion

A constant relation is revealed concerning exclusively to wave properties, not considered yet. It correlates with the electromagnetic coupling constant, which is currently inexplicable.

The obtained coincidence principally is possible to proof experimentally. That could confirm the wave origin of the electromagnetic coupling constant and wave-field nature of basic particles, as different kinds of quantum-wave formations.

The universality of α and its exposition in extremely large group of phenomena in microcosm becomes explainable; as a constant conditioned by wave-dynamic unique character of primordial substance (analogical π).

The absolute stability of α becomes clear, which means it is really "a constant"; it cannot vary with time as some researchers are inclined to see.

Proposed interpretation shows deep roots of wave-particle duality principle and its applicability in quantum electrodynamics level as well. It points on the unique nature of material world and on the possibility of unifying quantum and classical representations.

Presented approach and methodology may open an alternative way to study microcosm, empowering current research capabilities.

Methods

We have used the general conceptual principle and approach in our attempts to solve the examined problems, based on the unique wave-field nature of primordial substance.

Our method of analyses' is based on the geometric-imaginary representations and calculations allowing approximations. As an important criterion of trustfulness' of our approach we have looked at obtained series of known fundamental physical values, based on a unique concept.

We propose below described experiment as an independent confirmation of presented interpretation of the fine structure constant (see; "Results", chap. 1)

Proposed concept of Fine Structure Constant demands some correction to redistribution of interferential intensities. According to initial equations (2), we can obtain:

$$\Sigma I_m / I_0 = \tan\Delta\varphi \approx \sum_{m=1}^{n\to\infty} \frac{4}{\pi^2(2m+1)^2} \approx 0{,}094715. \qquad (63)$$

This value corresponds to $\Delta\varphi \approx 0.094433...$ that differs from (13). The task of experiments should be to define the actual value of $\Delta\varphi$, by the same to check the rightness of deduced results (13), (14). For such measurements we propose to use Fraunhofer's Single Slit Diffraction. The total intensity of the beam of light and intensity of main peak are necessary to establish in experiment, using photometric measurements with the same (P) photometer (or two calibrated ones) behind the slits S1, S2 (Fig. 10).

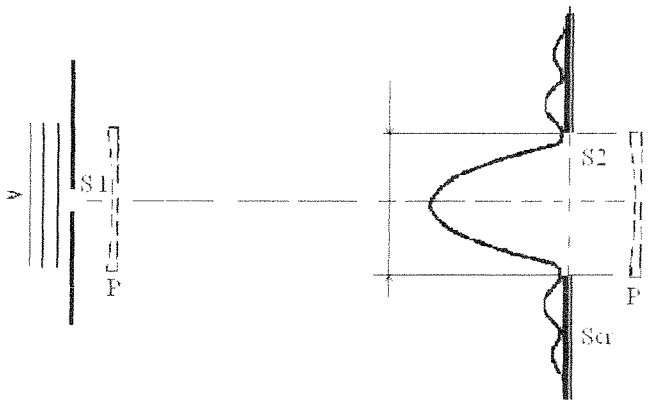

Figure 10. Fraunhofer's single slit diffraction

It is necessary to define the constant relation between its values. The exactness of results will mostly conditioned by exact coincidence of the sizes of the slit S2 with the displayed sizes of main peak of interference. (The direct measurement summary intensity of secondary peaks for a full angle of redistribution seems difficult from technical point).

By measured values I_0 , I we can define: $I_0 / I = cos\Delta\varphi$. In case (63), it has to be:

$$I_0 / I \approx 0.995544... \tag{64}$$

In case (13) it has should be:

$$I_0 / I = cos\,(arcsin\, e_*) \approx 0.996344... \tag{65}$$

The relative difference of two numbers is about $\delta \approx 8*10^{-4}$. The implemented collaboration laser-optics technique should to satisfy to mentioned conditions. The experiment will prove the wave origin of the "elementary charge" (e_*) as well as of the particle's mass.

Acknowledgment

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THE CHROMATIC POLYNOMIALS FOR THE WHEEL AND LADDER GRPAHS

Medhi Alaeiyan and Saeid Mohammadian
School of Mathematics
Iran University of Science and Technology
Narmak, Tehran 16844, Iran
(alaeiyan, s_mohammadian)@iust.ac.ir

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Abstract

Usually the coloring of the vertices of a graph G leads to discussion chromatic polynomial of graph, in which we have k distinct colors, (k is a nonnegative integer number), and we want to obtain the number of ways of coloring the vertices of G with k colors in such way that no two adjacent vertices are assigned the same color.This number of ways denoted by $P_G(k)$ and called chromatic function, since always this function is a polynomial so it called *chromatic polynomial.*
In this note we want to study on *cycle, wheel* and *ladder* graphs and obtain some ways to compute their chromatic polynomials.

Key words: Cycle graph; Wheel graph; Ladder graph; Chromatic number; Chromatic polynomial; Contracting an edge.

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1 Introduction

For entering to the argument we need some definitions and theorems that they are as follow:[1, 3, 4]

Definition 1.1 A *vertex coloring* of a graph is a vertex coloring such that the endpoints of each edge are assigned two different colors. A graph is said to be
vertex c-colorable if it has a vertex c-coloring, where c is a nonnegative integer number.

Definition 2.1 The *vertex chromatic number* of a graph G, denoted $\chi(G)$, is the minimum number of different colors required for a vertex-coloring of G. Thus, $\chi(G) = c$ if G is c-colorable but not $(c-1)$-colorable.

Definition 3.1 A graph G is *c-chromatic* if $\chi(G) = c$

Definition 4.1 Let G be a simple graph, and let $P_G(k)$ be the number of ways of coloring the vertices of G with k colors in such a way that no two adjacent vertices are assigned the same color. The function $P_G(k)$ is called the *Chromatic polynomial* of G, where k is a nonnegative integer number, and it is trivial that $k \geq \chi(G)$.

Theorem 5.1 *Let G be a simple graph, and let G_1 and G_2 be the graphs obtained from G by deleting and contracting an edge e respectively, then $P_G(k) = P_{G_1}(k) - P_{G_2}(k)$.*

Remark *If G be a simple graph, by contracting an edge e probably it consisted multiple edges, but the existence multiple edges doesn't influence on vertex coloring, so we dispense with this manner.*

2 Main results

At the first step we obtain a recursive relation for chromatic polynomial of cycle graph C_n. Let T_n be a tree with n vertices, one can see that:

$$P_{T_n}(k) = k(k-1)^{n-1},$$

also

$$P_{C_3}(k) = k(k-1)(k-2).$$

Now we consider the cycle C_4, by using Theorem 5.1 also by deleting and contracting an edge e we have a T_4 and a C_3, respectively, it follows that

$$\begin{aligned} P_{C_4}(k) &= P_{T_4}(k) - P_{C_3}(k), \\ &= k(k-1)^3 - k(k-1)(k-2) \\ &= k(k-1)(k^2-3k+3) \\ P_{C_4}(k) &= k(k-1)\{\binom{3}{0}k^2 - \binom{3}{1}k + \binom{3}{2}\}. \end{aligned}$$

Also for C_5, we have

$$\begin{aligned} P_{C_5}(k) &= P_{T_5}(k) - P_{C_4}(k), \\ &= k(k-1)^4 - k(k-1)(k^2-3k+3) \\ &= k(k-1)(k^3-4k^2+6k-4) \\ P_{C_5}(k) &= k(k-1)\{\binom{4}{0}k^3 - \binom{4}{1}k^2 + \binom{4}{2}k - \binom{4}{3}\}. \end{aligned}$$

By Theorem 5.1 and above argument we have the following lemma:

Lemma 1.2 *The chromatic polynomial of cycle graph C_n is as follow*

$$P_{C_n}(k) = P_{T_n}(k) - P_{C_{n-1}}(k), \quad where \quad n \geq 4,$$

or

$$P_{C_n}(k) = k(k-1)\{\binom{n-1}{0}k^{n-2} - \binom{n-1}{1}k^{n-3} + \cdots + (-1)^n\binom{n-1}{n-2}\}$$

$$P_{C_n}(k) = (k-1)\{\binom{n-1}{0}k^{n-1} - \binom{n-1}{1}k^{n-2} + \cdots + (-1)^n\}$$

$$P_{C_n}(k) = (k-1)^n + (-1)^n(k-1) \qquad \square.$$

Let us now turn to wheel graphs W_n that they obtained from C_{n-1} by joining each vertex to a new vertex x.
Let $G = (V, E)$ be a simple graph and $x \notin V$, the graph $G + x$ (or $G + K_1$) is given as (V_1, E_1) where $V_1 = V \cup \{x\}$ and $E_1 = E \cup \{e = vx; v \in V\}$.
So we have:

$$W_n = C_{n-1} + x,$$

and

Theorem 2.2 *Let G is a simple graph then*

$$P_{G+x}(k) = kP_G(k-1) \qquad \square.$$

by Theorem 2.2 we obtain the following Lemma for W_n.

Lemma 3.2

$$P_{W_n}(k) = kP_{C_{n-1}}(k-1) \quad where \quad n \geq 4$$

or

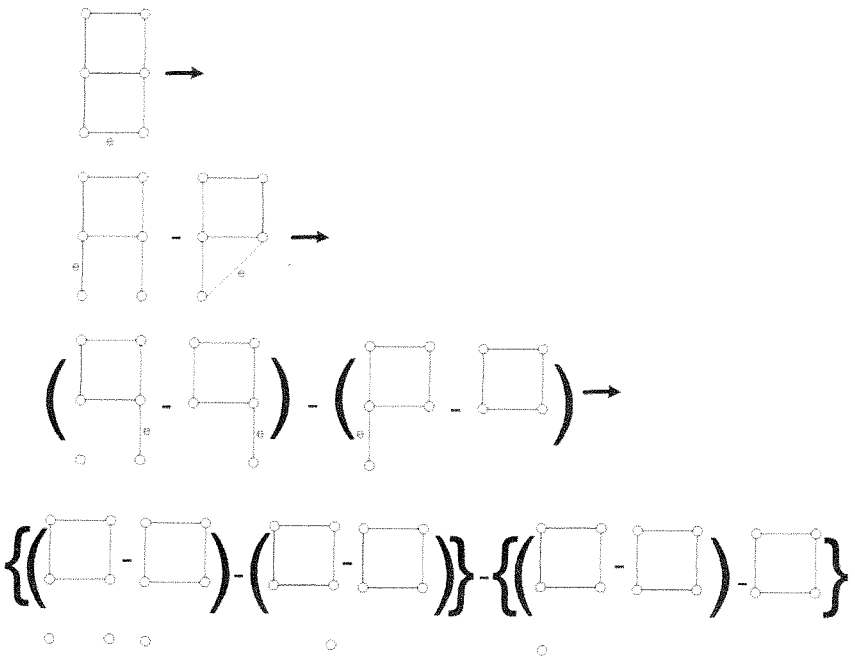
$$P_{W_n}(k) = kP_{C_{n-1}}(k-1) = k((k-1)(k-2)\{\binom{n-2}{0}(k-1)^{n-3} - \binom{n-2}{1}(k-$$

$1)^{n-4} + \cdots + (-1)^{n-1}\binom{n-2}{n-3}\})$ $\square$.

Now we compute the chromatic polynomial for ladder graphs L_n, that $L_2 = C_4$ and L_n, $n \geq 3$, can be viewed as the cartesian product $P_2 \times P_n$ of a path on two vertices and a path on n vertices.

$$P_{L_2}(k) = P_{C_4}(k) = (k^4 - 4k^3 + 6k^2 - 3k)$$

With the same argument about C_n, by deleting and contracting an edge e respectively from L_n, we have the following figure



$$P_{L_3}(k) = (k^2 - 3k + 3)P_{L_2}(k).$$

So

$$P_{L_3}(k) = (k^2 - 3k + 3)P_{L_2}(k).$$

With the same process on L_4 we have

$$P_{L_4}(k) = (k^2 - 3k + 3)P_{L_3}(k).$$

or

$$P_{L_4}(k) = (k^2 - 3k + 3)^2 P_{L_2}(k).$$

So by the above argument we have the following Lemma:

Lemma 4.2

$$P_{L_n}(k) = (k^2-3k+3)^{n-2}P_{L_2}(k), \quad where \quad n > 2,$$

$$P_{L_2}(k) = (k^4 - 4k^3 + 6k^2 - 3k).$$

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TO CONFIRM THE EXISTENCE OF ATOMIC GRAVITATIONAL CONSTANT

U. V. S. Seshavatharam
Honorary Faculty, I-SERVE
Alakapuri, Hyderabad-35, India
seshavatharam.uvs@gmail.com
Prof. S. Lakshminarayana
Department of Nuclear Physics
Andhra University
Visakhapatnam-03, India
lnsrirama@yahoo.com

Abstract

In the atomic or nuclear space, till today no one measured the value of the gravitational constant. To bring down the planck mass scale to the observed elementary particles mass scale a large scale factor is required. Ratio of planck mass and electron mass is close to Avogadro number/$8\pi \cong N/8\pi$. The idea of strong gravity originally referred specifically to mathematical approach of Abdus Salam of unification of gravity and quantum chromo-dynamics, but is now often used for any particle level gravity approach. In this connection it is suggested that ratio of atomic gravitational constant (G_A) and classical gravitational constant (G_C) is equal to N^2. If $R_0 \cong 1.21$ fermi is the nuclear charge radius, to a very good accuracy it is noticed that in Hydrogen atom, ratio of total energy of electron and nuclear potential is equal to the electromagnetic and gravitational force ratio of electron where the operating gravitational constant is N^2G_C but not G_C. Similar to the classical force limit $\frac{c^4}{G}$, nuclear weak force magnitude can be expressed as $\frac{c^4}{N^2G} \cong \frac{c^4}{G_A}$. This

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is another significance of Avogadro number. Square root of ratio of strong and weak force magnitudes can be expressed as $2\pi \ln(N^2)$. With the defined strong and weak force magnitudes observed elementary particles masses and their magnetic moments can be generated. Interesting application is that: characteristic building block of the cosmological 'dark matter' can be quantified in terms of fundamental physical constants. No extra dimensions are required in this new approach.

Keywords: Classical gravitational constant, atomic gravitational constant, Avogadro number, grand unification, dark matter, electron rest mass, gram mole, Hydrogen atom, classical force limit, weak force, strong force, proton rest mass, fine structure ratio, Fermi's weak coupling constant, strong coupling constant, electro weak energy scale, semi empirical mass formula.

1 Introduction

As the culmination of his life work, Einstein wished to see a unification of gravity and electromagnetism as aspects of one single force. In modern language he wished to unite electric charge with the gravitational charge (mass) into one single entity. Further, having shown that mass the gravitational charge was connected with space-time curvature, he hoped that the electric charge would likewise be so connected with some other geometrical property of space-time structure. To unify 2 interactions if 5 dimensions are required, for unifying 4 interactions 10 dimensions are required. For 3+1 dimensions if there exists 4 (observed) interactions, for 10 dimensions there may exist 10 (observable) interactions. To unify 10 interactions 20 dimensions are required. This logic seems to indicate that with 'n' new dimensions on may not be able to resolve the problem of unification. More over new problems and new properties will come into picture and makes the 4 dimensional unification program more complicated. Right now quantitatively and qualitatively: 1) one can not implement the planck scale in 'atomic' and 'nuclear' space. 2) one can not think about the 'reduced magnitudes' of quantized elementary charge or angular momentum. The only

one simple alternative for understanding 'unification' is - to think about the 'variation of gravitational constant' or to think about the existence of 'atomic gravitational constant'. Magnitude and existence of the proposed atomic gravitational constant may be 'absolute' or 'relative'. Its existence has to be confirmed by logical analysis. Interesting application is: it helps in finding the characteristic building block of the 'cosmological dark matter'.

2 Variation of the classical gravitational constant in cosmology

In understanding the cosmic evolution or the large scale structure of the universe, in his large number hypothesis, Dirac assumed that magnitude of the gravitational constant is inversely proportional to the cosmic time. At the same time he assumed that in the past there was no change in the magnitude of atomic physical constants. J.V.Narlikar in his book explained it in detail [1].

$$G \propto \frac{1}{t}. \tag{1}$$

But cosmic variation of G goes against the concepts of general theory of relativity. Sciama assumed that there exists a relation between inertia and the large scale structure of universe as

$$G \cong \frac{Rc^2}{M}. \tag{2}$$

Brans and Dicke postulated that G behaves as a reciprocal of a scalar field as

$$G \cong \frac{1}{\Phi}. \tag{3}$$

where Φ is expected to satisfy a scalar wave equation whose source is all the matter in the universe.

Equations (1), (2) and (3) suggests that theoretically it is possible to think about the variation of G. Whether the nature of variation is cosmic or there exists two kinds of gravitational constants one for the classical physics and the other for the atomic system- has to be analysed. From

modern flat model of cosmology point of view there is no need to consider a variable cosmic gravitational constant. Seshavatharam [2, 3] proposed a unified model of black hole cosmology. From this also it is clear that for understanding the cosmic evolution there is no need to consider the Dirac's view of variable G. In this paper this second idea is discussed. Authors humbly say- this concept can successfully be applied in the unification of the four fundamental interactions including the cosmic dark matter. Note that in the atomic or nuclear physics, till today no one measured the gravitational force of attraction between the proton and electron and experimentally no one measured the value of the gravitational constant. Physicists say - if strength of strong interaction is unity, with reference to the strong interaction, strength of gravitation is 10^{-39}. The fundamental question to be answered is: is mass an inherent property of any elementary particle?

Authors humbly say: for any elementary particle mass is an induced property. This idea makes grand unification easy. Hawking S.W, Abdus Salam, David Gross and Tilman sauer presented a beautiful discussion on Unification ([4],[5],[6],[7]). Note that GTR does not throw any light on the mass generation of charged particles. It only suggests that space-time is curved near the massive celestial objects. More over it couples the cosmic (dust) matter with geometry. But how matter is created? Why and how elementary particle possesses both charge and mass? Such type of questions are not discussed in the frame work of GTR.

It is well known that celestial bodies constitutes so may electrons and nucleons. Clearly speaking mass of the celestial body is an index of how many nucleons it constitutes. The subject of unification is broad in the sense it makes an attempt to understand the origin of 'mass generation' of elementary particles. In this situation one can confidently say - the existing gravitational constant is a consequence of grand unification. Not only that the mysterious concept 'gram mole' can be understood very easily. In Dirac's model G is large in the past and works for the construction of the universe where as in the charge-mass unification program grand unified G is large and works for the construction of the massive elementary particles.

3 Grand unification and the need of atomic gravitational constant

The strong or atomic gravitational constant is the supposed physical constant of strong gravitation, involved in the calculation of the gravitational attraction at the level of elementary particles and atoms. The idea of strong gravity originally referred specifically to mathematical approach of Abdus Salam [8,9,10] of unification of gravity and quantum chromo-dynamics, but is now often used for any particle level gravity approach. In literature one can refer the works of Abdus Salam, C. Sivaram, Sabbata, A.H. Chamseddine, J. Strathdee, Usha Raut, K. P. Sinha, J.J.Perng, E. Recami, R. L. Oldershaw, K.Tennakone, S.I Fisenko and S.G.Fedosion ([11]-[24]).

From the standpoint of 'infinite hierarchical nesting of matter' and Le Sage's theory of gravitation, the presence of two gravitational constants shows the difference between the properties of gravitons and properties of matter at different levels of matter. The strong gravitational constant is also included in the formula describing the nuclear force through strong gravitation and torsion field of rotating particles. A feature of the gravitational induction is that if two bodies rotate along one axis and come close by the force of gravitation, then these bodies will increase the angular velocity of its rotation. In this regard, it is assumed that the nucleons in atomic nuclei rotate at maximum speed. This may explain the equilibrium of the nucleons in atomic nuclei as a balance between the attractive force of strong gravitation and the strong force of the torsion field (of gravito-magnetic forces in gravito-magnetism). Various proposed values of strong gravitational constant are 2.06×10^{25}, 6.7×10^{27}, 2.18×10^{28}, 2.4×10^{28}, 3.9×10^{28}, 1.514×10^{29}, 3.2×10^{30}, 5.1×10^{31}, 6.9×10^{31} , $2.77\times10^{32}\ m^3Kg^{-1}sec^{-2}$.

The subject of unification is not new. For Einstein - the existence, the mass, the charge of the electron and the proton, the only elementary particles recognized back in the 1920s, were arbitrary features. One of the main goals of a unified theory should be to explain the existence and calculate the properties of matter. But here the fundamental question to be answered is: without a 'mass content' can electric charge preserve its individual identity? For example even though 'rest mass' of photon is zero it possesses 'energy'. For any elementary charged massive particle - which is more fundamental

either the 'mass' or the 'charge'? Here authors humble opinion is : charge can be considered as the fundamental, inherent and characteristic property of the charged massive particle. For the same magnitude of charge, proton's mass is 1836.15 times heaver than the mass of electron. Observed elementary mass spectrum ranges from 0.511 MeV to 182 GeV. But very interesting and surprising observation is that magnitude of charge remains at e or $2e$. How to understand this situation? Concept of quantization of charge states that- in nature 'charge' exists only in integral multiples of e.

Stephen Hawking - in his famous book- says: It would be very difficult to construct a complete unified theory of everything in the universe all at one go. So instead we have made progress by finding partial theories that describe a limited range of happenings and by neglecting other effects or approximating them by certain numbers. (Chemistry, for example, allows us to calculate the interactions of atoms, without knowing the internal structure of an atomic nucleus.) Ultimately, however, one would hope to find a complete, consistent, unified theory that would include all these partial theories as approximations, and that did not need to be adjusted to fit the facts by picking the values of certain arbitrary numbers in the theory. The quest for such a theory is known as "the unification of physics". Einstein spent most of his later years unsuccessfully searching for a unified theory, but the time was not ripe: there were partial theories for gravity and the electromagnetic force, but very little was known about the nuclear forces. Moreover, Einstein refused to believe in the reality of quantum mechanics, despite the important role he had played in its development.

The first step in unification is to understand the origin of the rest mass of a charged elementary particle. Second step is to understand the combined effects of its electromagnetic (or charged) and gravitational interactions. Third step is to understand its behaviour with surroundings when it is created. Fourth step is to understand its behaviour with cosmic space-time or other particles. Right from its birth to death, in all these steps the underlying fact is that whether it is a strongly interacting particle or weakly interacting particle, it is having some rest mass. To understand the first 2 steps somehow one must implement the gravitational constant in sub atomic physics. Seshavatharam and Lakshminarayana [25-29] proposed

that there may exist coulomb's charged particle of mass-energy

$$M_c c^2 \cong \sqrt{\frac{e^2}{4\pi\epsilon_0}\left(\frac{c^4}{G}\right)} \cong 1.042940852 \times 10^{18}\ GeV. \tag{4}$$

Recalling Einstein's view that 'unification of gravity and electromagnetism as aspects of one single force', considering the classical limit of force $\frac{c^4}{G}$ -its large magnitude play some interesting role in grand unification and general theory of relativity. Seshavatharam [2,3], W.C.Daywitt [30] and N. Harameian [31] discussed its role in Black hole physics, Planck scale physics and General theory of Relativity receptively.

$$M_c \cong \sqrt{\frac{e^2}{4\pi\epsilon_0 G}} \cong 1.859210775 \times 10^{-9}\ Kg. \tag{5}$$

The beauty of this expression is that it generates a 'mass content' from e and G. In the sense it is generating 'inertia' in the free space. Here the fundamental questions to be answered are: from where elementary charge is coming into picture? How and why it exists in the universe? How many elementary charges are there in the universe? Is 'coulomb mass' the mother of all the observed charged and neutral elementary massive particles? Qualitatively this obtained mass unit play some role in the generation of elementary particle's rest mass. But from numerical point of view this mass is very large compared to the observed elementary particle's rest mass. Till today in the laboratory no such a particle is observed with such a large mass. To move from this large mass unit to the electron mass one must consider some type of large coupling constant or a proportionality number or a scaling factor. Now the real problem comes into picture. If the scaling factor or proportionality number is a known one, then to some extent - its historical data and physics background makes and brings the unification concepts into one stream. Compared to the current research - it may be in the main streamline or secondary streamline - it can be decided by the future thoughts and experiments.

To have a small mass unit one cannot assume that small massive particle possesses a fractional magnitude of e. In CGS system of units value of $4\pi\epsilon_0$ is unity. The only one alternative that must be allowed is - variation of G.

Inserting a coupling number or proportionality number means- it may be a system constant or interaction constant. The main object of unification is to understand the relation between the three atomic interactions and the gravitational interaction. Ultimately one must co-relate the coupling constant with the gravitational constant. Finally this leads to the concept of the variation of G. For each and every elementary particle its corresponding value of G can be expressed as

$$G_m \cong \frac{e^2}{4\pi\epsilon_0 m_x^2}. \tag{6}$$

Here G_m = magnitude of G corresponding to the mass of the particle m_x. The interesting point to be noted is that unlike the classical or continuous mass range of celestial massive bodies, elementary particles mass spectrum follows certain quantum rules and hence there exists some governing procedure for the observed mass spectrum. Not only that each interaction is having some coupling constants. Considering leptons three exists only one basic particle- that is electron. Considering hadrons there exists only one stable particle - that is proton. Hence value of G_m can be fixed. If one is able to inter change the coupling constants , there is a possibility of fixing the value of G_m. In this way this proposed idea differs from Dirac's proposal of variation of G with cosmic time. Based on Sciama's proposal, in atomic and nuclear physics, with reference to the nuclear mass and size, magnitude of the nuclear characteristic gravitational constant can be given as

$$G_m \cong \frac{R_p c^2}{m_p}. \tag{7}$$

Here, m_p = mass of proton, R_p = size of proton.

To bring down the planck mass scale to the observed elementary particles mass scale a large scale factor is required. Just like relative permeability and relative permittivity by any suitable reason in atomic space or nuclear space if one is able to increase the value of classical gravitational constant, it helps in four ways. Observed elementary particles mass can be generated and grand unification can be achieved. Third important application is characteristic building block of the cosmological 'dark matter' can be quantified in terms of fundamental physical constants. Fourth important application

is - no extra dimensions are required. Finally nuclear physics and quantum mechanics can be studied in the view of 'strong nuclear gravity' where nuclear charge and atomic gravitational constant play a crucial role in the nuclear space-time curvature, QCD and quark confinement. Not only that cosmology and particle physics can be studied in a unified way.

In this connection it is suggested that square root of ratio of atomic gravitational constant and classical gravitational is equal to the Avogadro number. Till today there is no explanation for this fantastic large number. It is an observed fact. The very unfortunate thing is that even though it is a large number it is neither implemented in cosmology nor implemented in grand unification. Note that ratio of planck mass and electron mass is close to $\frac{N}{8\pi}$. The Avogadro constant expresses the number of elementary entities per mole of substance. Avogadro's constant is a scaling factor between macroscopic and microscopic (atomic scale) observations of nature.

It can be supposed that elementary particles construction is much more fundamental than the black hole's construction. If one wishes to unify electroweak, strong and gravitational interactions it is a must to implement the classical gravitational constant G in the sub atomic physics. By any reason if one implements the planck scale in elementary particle physics and nuclear physics automatically G comes into subatomic physics. Then a large arbitrary number has to be considered as a proportionality constant. After that its physical significance has to be analysed. Alternatively its equivalent 'strong atomic gravitational constant' can also be assumed. Some attempts have been done in physics history.

Whether it may be real or an equivalent if it is existing as a 'single constant' its physical significance can be understood. 'Nuclear size' can be fitted with 'nuclear Schwarzschild radius'. 'Nucleus' can be considered as 'a strong nuclear black hole'. This idea requires a basic nuclear fermion! Nuclear binding energy constants can be generated directly. Proton-neutron stability can be studied. Origin of 'strong coupling constant' and 'Fermi's weak coupling constant' can be understood. Charged lepton masses can be fitted. Authors feel that these applications can be considered favourable for the proposed assumptions and further analysis can be carried out positively for understanding and developing this proposed 'Avogadro's strong nuclear gravity'.

4 Planck scale charged space-time curvature

A characteristic planck scale space-time curvature can be expressed as

$$R_P \cong \sqrt{\frac{e^2}{4\pi\epsilon_0}\left(\frac{G}{c^4}\right)} \cong 1.380677 \times 10^{-36}\ m. \tag{8}$$

General theory of relativity says that space-time is curved at massive bodies surface. Coming to the microscopic physics there exists only one elementary charge. Above expression indicates that under certain unknown and extreme physical conditions space-time is curved near the surface of a charged elementary particle. Here also by inserting a coupling constant or proportionality ratio this small length can be increased to the observed elementary particle's characteristic size. In a reverse way for each and every elementary particle's size its corresponding value of G_R can be expressed as

$$G_R \cong \frac{4\pi\epsilon_0 R_x^2 c^4}{e^2}. \tag{9}$$

where R_x is the size of any elementary particle. Present day experimental physics suggests that leptons has no structure yet all. It means leptons are point particles and there is no definite size. The remaining particles are only stable hadrons or the atomic nucleus. Rutherford's α-scattering experiments suggests that there exists a characteristic nuclear unit size. Hence the value of G_R can be fixed based on the nuclear size.

Out of the four observed fundamental interactions, there exists some similarities in between the electromagnetic and gravitational interactions. Both are long range forces and follows inverse square law. In both of these cases field carriers move with speed of light. Grand unification program suggests that in the past during the cosmic evolution all the four interactions are same and possesses same strength. Distance being the same, ratio of electromagnetic force and gravitational force between proton and electron is always

$$r_{eg} \cong \frac{e^2}{4\pi\epsilon_0 G m_p m_e}. \tag{10}$$

Here m_p =mass of proton, m_e=mass of electron, e = charge on electron or proton and r_{eg} = electromagnetic and gravitational force ratio. Above idea can be represented as in the past

$$\frac{e^2}{4\pi\epsilon_0 d_e^2} \cong \frac{c^4}{G}. \tag{11}$$

$$\frac{Gm_p m_e}{d_g^2} \cong \frac{c^4}{G}. \tag{12}$$

$$\frac{d_e}{d_g} \cong \sqrt{\frac{e^2}{4\pi\epsilon_0 G m_p m_e}}. \tag{13}$$

Here d_e=distance between electron and proton as 2 charges and d_g = distance between electron and proton as 2 massive particles.

5 Atomic gravitational constant and mystery of dark matter candidate

Modern cosmological observations and analysis clearly suggests that there exists a mysterious matter which can be called as'dark matter' and 'dark energy'. A.J. Frieman, M.S. Turner and D.Huterer explained this in their paper [32]. Please note that existence of 'dark matter' is not proposed from particle physics observations. This is a very strange idea. If one is able to find the 'dark matter' it will be an indication of 'unification of the fundamental interactions'. At the beginning 'dark matter' existence is guessed from rotational velocities of stars, gas clouds, globular clusters and satellite dwarf galaxies at the periphery of galaxies. In General Relativity, for the universe, accelerated expansion is possible only if there exists 'dark energy'. There are several other, independent arguments based, in particular, on the estimate of the age of the Universe, structure formation, cluster abundance, CMB anisotropy. All of them point to the existence of 'dark energy' whose density today is at the level of 75% of the critical density.

Please note that till today in modern physics history there is no clear cut mechanism or expression for the generation of dark matter 'mass'. Which

particles make non-baryonic clustered dark matter is not known experimentally. One expects that these are stable or almost stable particles that do not exist in the Standard Model of particle physics. Hence, the very existence of dark matter is a very strong argument for incompleteness of the Standard Model. This makes the detection and experimental study of the dark matter particle extremely interesting and important. On the other hand, the lack of experimental information on the properties of these particles makes it impossible to give a unique answer to the question of the mechanism of the dark matter generation in the early Universe.

It is noticed that ratio of planck mass and electron mass is 2.389×10^{22} and is 25.2 times smaller than the Avogadro number. It is also noticed that the number 25.2 is close to 8π. Qualitatively this idea implements gravitational constant in particle physics. Note that planck mass is the heaviest mass and neutrino mass is the lightest mass in the known elementary particle mass spectrum. As the mass of neutrino is smaller than the electron mass, ratio of planck mass and neutrino mass will be close to the Avogadro number or crosses the Avogadro number. Since neutrino is an electrically neutral particle if one is able to assume a charged particle close to neutrino mass it opens a window to understand the combined effects of electromagnetic (or charged) and gravitational interactions in sub atomic physics. Compared to planck scale (past cosmic high energy scale), Avogadro number is having some physical significance in the (observed or present low energy scale) fundamental physics or chemistry.

$$\frac{M_P}{m_e} \cong \sqrt{\frac{\hbar c}{G m_e^2}} \cong 2.3892245954 \times 10^{22} \cong \frac{N}{8\pi}. \tag{14}$$

Here, M_P = planck mass and m_e = electron rest mass. Hence electron rest mass can be expressed as

$$m_e \cong \frac{8\pi}{N}\sqrt{\frac{\hbar c}{G}} \cong 8\pi\sqrt{\frac{\hbar c}{N^2 G}} \cong 9.083115709 \times 10^{-31}\ Kg. \tag{15}$$

Accepted value of $m_e = 9.109382154 \times 10^{-31}\ kg$ and accuracy is 99.7116%. In terms of the above introduced 'coulomb' mass unit it can be expressed as

$$m_e \cong \frac{8\pi}{N\sqrt{\alpha}}\sqrt{\frac{e^2}{4\pi\epsilon_0 G}} \cong \frac{8\pi}{\sqrt{\alpha}}\sqrt{\frac{e^2}{4\pi\epsilon_0\left(N^2G\right)}}. \tag{16}$$

Here it can be assumed that- if $\frac{8\pi}{\sqrt{\alpha}} \cong 294.2098$ is the electromagnetic 'mass induction strength' or 'mass generation strength' then $N^2G \cong G_A$ can be considered as the atomic gravitational constant. In grand unification program this number

$$X_E \cong \frac{8\pi}{\sqrt{\alpha}} \cong \sqrt{\frac{4\pi\epsilon_0\left(N^2G\right)m_e^2}{e^2}} \cong 295.0606338. \tag{17}$$

can be called as the lepton-quark-nucleon gravitational mass generator. It is the utmost fundamental ratio compared to the fine structure ratio α. It is noticed that

$$\frac{1}{\alpha} \cong \frac{1}{2}\sqrt{X_E^2 - \left[\ln\left(N^2\right)\right]^2} \cong 136.9930484. \tag{18}$$

This can be compared with 'inverse of the fine structure ratio' $= \frac{1}{\alpha} \cong 137.0359997$. Another strange observation is that

$$\ln\left(\frac{M_c}{m_p}\right) \cong \ln\sqrt{\frac{e^2}{4\pi\epsilon_0 G m_p^2}} \cong \sqrt{\frac{m_p}{m_e} - \ln\left(N^2\right)}. \tag{19}$$

Here $M_c \cong \sqrt{\frac{e^2}{4\pi\epsilon_0 G}}$, m_p is the proton rest mass and m_e is the electron rest mass. N is the Avogadro number and G is the gravitational constant. $\frac{e^2}{4\pi\epsilon_0 G m_p^2}$ is the electromagnetic and gravitational force ratio of proton. Carlo Amedeo Avogadro [33] proposed his famous hypothesis in 1811. P.J. Mohar and B.N.Taylor [34] recommended values are $N = 6.022141793 \times 10^{23}$ and $G_C \cong 6.6742867 \times 10^{-11}\ m^3Kg^{-1}sec^{-2}$. Here in this equation (19), Lhs = 41.55229152; Rhs = 41.55289244; This is an excellent fit. In grand unification program this type of fitting should not be ignored. This relation clearly suggests that there exists a definite relation between M_c, m_p, m_e and N^2. Considering all the atomic physical constants, obtained value of the gravitational constant is $6.666270179 \times 10^{-11}\ m^3Kg^{-1}sec^{-2}$. Till today

no atomic model implemented the gravitational constant in the atomic or nuclear physics. Then, whatever may be its magnitude, measuring its value from existing atomic principles is impossible. Its value was measured in the lab within a range of 1 cm to 1 meter only where as the observed nuclear size is 1.2 fermi. With reference to the above relations it is possible to define a new mass unit as

$$m_X \cong \sqrt{\frac{e^2}{4\pi\epsilon_0 \left(N^2 G\right)}} \cong 3.087291597 \times 10^{-33}\ Kg. \tag{20}$$

$$m_X c^2 \cong \sqrt{\frac{e^2 c^4}{4\pi\epsilon_0 \left(N^2 G\right)}} \cong \sqrt{\frac{e^2}{4\pi\epsilon_0}\left(\frac{c^4}{N^2 G}\right)} \cong 1.731843735\ KeV. \tag{21}$$

This mass unit is very close the (neutral) neutrino mass. Conceptually this can be compared with the 'charged' dark matter. Note that either in cosmology or particle physics till today there is no clear cut mechanism for understanding the massive origin of the dark matter. Its existence changes the fate of 'modern' thoughts in cosmology and particle physics. In this critical situation proposed ideas can be given a chance.

The fundamental question to be answered is : 1.7318 KeV is a potential or a charged massive particle? If it is a particle its pair annihilation leads to radiation energy. If it is the base particle in elementary particle physics - observed particle rest masses can be fitted. Authors humble opinion is: it can be considered as the basic charged lepton or lepton potential. It can be considered as the basic charged 'dark matter' candidate. Magnitude of $G_A \cong N^2 G = 2.420509614 \times 10^{37}\ m^3 kg^{-1} sec^{-2}$.

At this moment a modern physicist cannot admit this idea. Their view is that this large value of G cannot be incorporated in the GTR or existing physics. This is absolutely true. Absolute lab measurements of G have been made on the scales of about 1 cm to 1 meter only. For any experimental physicist it is a must to measure the magnitude of G in nuclear physics. Without measuring its value how can one say that the same value of G operates in the atomic or nuclear space time curvature. Here the very important question to be answered is: which is more fundamental either G or $N^2 G$? Authors humble opinion is : both can be considered as the 'head' and 'tail' of matter coin. It can also be suggested that G is a consequence

of the existence of N^2G. Please note that even if human beings are able to understand the 'absolute findings' they may not be able to make the 'absolute measurements'. In terms of Planck mass its neutral mass unit can be represented as

$$m_P \cong \sqrt{\frac{\hbar c}{(N^2G)}} \cong 3.614056909 \times 10^{-32}\ Kg. \tag{22}$$

$$m_P c^2 \cong \sqrt{\frac{\hbar c^5}{(N^2G)}} \cong \sqrt{\hbar c \left(\frac{c^4}{N^2G}\right)} \cong 20.27337431\ KeV. \tag{23}$$

Considering these expressions for dark matter and with a suitable theoretical model along with a suitable proportionality ratio like the fine structure ratio or the strong coupling constant correct magnitude of dark matter mass unit can be estimated. Most interesting thing is that unless one consider the 'atomic gravitational constant'= $N^2G = G_A$ this is not possible.

5.1 Charged lepton rest masses

Let G_C = gravitational constant operating in the free space or universe, N = Avogadro number and G_A= gravitational constant operating in the atomic and nuclear system= N^2G_C.

Using the above defined number $X_E = 295.0606338$, charged lepton masses can be fitted as

$$m_l c^2 \cong \left[X_E^3 + \left(n^2 X_E\right)^n \sqrt{N}\right]^{\frac{1}{3}} \sqrt{\frac{e^2c^4}{4\pi\epsilon_0 G_A}} \cong \frac{2}{3}\left[E_c^3 + \left(n^2 X_E\right)^n E_a^3\right]^{\frac{1}{3}}. \tag{24}$$

Here n= 0,1, 2. E_c and E_a are the coulombic and asymmetric energy constants of the semi empirical mass formula. Qualitatively this expression is connected with β decay. See the following table-1. Please refer M.Yao et al [35] recommended PDG charged lepton masses. If electron mass is fitting at n = 0, muon mass is fitting at n = 1 and tau mass is fitting at n = 2 it is quite reasonable and natural to predict a new heavy charged lepton at n = 3. By selecting the proper quantum mechanical rules if one is able to confirm the existence of the number n = 3, existence of the new lepton can

n	Obtained Lepton mass, MeV	Exp. Lepton Mass, MeV
0	Defined	0.510998922
1	105.951	105.658369
2	1777.384	1776.84 $\pm$0.17
3	42262.415	to be discovered

Table 1: Fitting of charged lepton rest masses.

be understood. Recent experiments suggests that there exists a 4th flavour neutrino. At n=3 there may exist a heavy charged lepton at 42262 MeV. At this moment one can not deny this prediction. At the same time one must critically examine the proposed relation for its nice and accurate fitting of the 3 observed charged leptons. Unfortunately inputs of this expression are new for the standard model. Hence one can not easily incorporate this expression in standard model. Till now in SM there is no formula for fitting the lepton masses accurately. It indicates the incompleteness of the SM.

5.2 Mystery of the gram mole

Authors humble opinion is - Avogadro number is not a pure number. Clearly speaking it is the square root of ratio of strong nuclear gravitational constant and the classical gravitational constant.

$$N \cong \sqrt{\frac{G_A}{G_C}}. \tag{25}$$

In SI system of units why gram mole is being used? This fundamental question can be answered if it is assumed that there exists a limit for the quantum mechanical atomic mass. The definition of 'quantum mechanical atomic mass' can be given as- it is the upper limit for the mass of an elementary particle or mass of a microscopic system or mass of an atom where in the existing quantum mechanical and atomic laws can be applied. If mass of the system crosses the limit, quantum mechanics and atomic structure transforms to classical physical laws. Quantitatively the assumed mass limit can be obtained in the following way.

$$G_A m_p^2 \cong G_C M_X^2. \tag{26}$$

where m_p = operating mass unit in atomic physics = mass of proton and M_X = operating mass unit in classical physics.

$$\left(\frac{M_X}{m_p}\right)^2 \cong N^2. \tag{27}$$

Hence $M_X \cong N \times m_p \cong 1.0072466 \times 10^{-3}\ Kg \cong 1.0072466\ gram$. In this way gram mole can be understood.

6 Nuclear charge radius, atomic gravitational constant and the Hydrogen atom

In 1911 under the supervision of Rutherford, H.Giger and E.Marseden [36] for the first time experimentally showed that nuclear size is the order of 1.4 fermi. Later electron scattering experiments revealed that at a distance of R_0 from the nuclear center nuclear charge density falls to 50% of its maximum charge density. If $R_0 \cong 1.21$ fermi is the nuclear charge radius, to a very good accuracy it is noticed that in Hydrogen atom, ratio of total energy of electron and nuclear potential is equal to the electromagnetic and gravitational force ratio of electron where the operating gravitational constant is $N^2 G_C$ but not G_C. With reference to Bohr's theory of Hydrogen atom, it can be expressed as

$$-\frac{e^2}{8\pi\epsilon_0 a_0} \cong -\frac{e^2}{4\pi\epsilon_0 G_A m_e^2} \times \frac{e^2}{4\pi\epsilon_0 R_0}. \tag{28}$$

Here a_0 is the bohr radius of electron in Hydrogen atom and R_0 is the nuclear charge radius. This expression clearly confirms the existence of the $G_A \cong N^2 G_C$ in atomic physics.

$$-\frac{e^2}{8\pi\epsilon_0 a_0} \cong -\frac{1}{X_E^2} \times \frac{e^2}{4\pi\epsilon_0 R_0}. \tag{29}$$

$$X_E^2 \cong \frac{e^2}{4\pi\epsilon_0 G_A m_e^2}, \tag{30}$$

can be considered as the ratio of electromagnetic and gravitational forces of electron where the operating gravitational constant is N^2G_C but not G_C.

$$a_0 \cong \frac{4\pi\epsilon_0 G_A m_e^2}{2e^2} \times R_0. \tag{31}$$

$$\frac{a_0}{R_0} \cong \frac{4\pi\epsilon_0 G_A m_e^2}{2e^2} \cong \frac{X_E^2}{2}. \tag{32}$$

Revolving electrons basic angular momentum can be expressed as

$$m_e v r \cong \sqrt{\frac{N^2 G_C\, m_e^3\, R_0}{2}} \cong \hbar \cong N\sqrt{\frac{G_C\, m_e^3\, R_0}{2}}. \tag{33}$$

where r is the orbit radius and v orbiting speed. The most important observation is: in atomic physics there exists a grand unified angular momentum and can be expressed as

$$\frac{\hbar}{N} \cong \frac{h}{2\pi N} \cong \sqrt{\frac{G_C\, m_e^3\, R_0}{2}}. \tag{34}$$

where h is the famous planck's constant. The basic quanta of angular momentum is N times of $\sqrt{\frac{G_C\, m_e^3\, R_0}{2}}$. This is a very strange concept that couples the micro-macro physical constants. This can be considered as another definition to the Avogadro number. One cannot deny the existence of N^2G_C in the grand unification program. This may be considered as the origin of quantum mechanics. The fundamental question to be answered is: In understanding the energy spectrum of Hydrogen atom out of R_0 and $\hbar$ which is the primary physical constant?

$$v r \cong \sqrt{\frac{N^2 G_c\, m_e\, R_0}{2}} \cong \frac{\hbar}{m_e} \cong N\sqrt{\frac{G\, m_e\, R_0}{2}}. \tag{35}$$

Guessing that quantum mechanics play a vital role in nuclear physics, nuclear charge radius can be expressed as

$$R_0 \cong \frac{1}{N^2}\left(\frac{\hbar c}{G_C m_e^2}\right)^2 \frac{2G_C m_e}{c^2} \cong \frac{2\hbar^2}{G_A m_e^3} \cong 1.215650083\ fermi. \tag{36}$$

Here m_e is the rest mass of electron and $\frac{2G_C m_e}{c^2}$ is nothing but the classical black hole radius of electron.

$$N^2 \cong \frac{2\hbar^2}{G_C m_e^3 R_0}. \tag{37}$$

$$G_A \cong N^2 G_C \cong \frac{2\hbar^2}{m_e^3 R_0} \cong \left(\frac{\hbar}{m_e c R_0}\right)^2 \frac{2R_0 c^2}{m_e}. \tag{38}$$

Qualitatively this idea represents the Sciama's idea of inertia of the large scale massive universe. Using this incredible expression value of $N^2 G_C$ can be estimated. If Avogadro number is known, value of G_C can be directly estimated from the atomic physical constants accurately.

$$G_C \cong \frac{2\hbar^2}{N^2 m_e^3 R_0} \cong \left(\frac{\hbar}{N m_e c R_0}\right)^2 \frac{2R_0 c^2}{m_e}. \tag{39}$$

Accuracy depends only on the value of R_0. But till today its origin is a mystery.

7 The nuclear weak force and strong force magnitudes

In classical physics or in cosmology or in black hole physics or in planck scale physics, the operating 'classical force limit' is $F_C \cong \left(\frac{c^4}{G_C}\right)$. Similar to this, the characteristic force limit in atomic or nuclear physics can be given as $\left(\frac{c^4}{G_A}\right)$. It can be expressed as

$$\frac{c^4}{G_A} \cong \frac{c^4}{N^2 G_C} \cong 3.337152088 \times 10^{-4}\ newton. \tag{40}$$

The most surprising observation is that this force magnitude can be termed as the 'nuclear weak force constant' and can be represented as

$$F_W = \frac{c^4}{G_A} \cong \frac{1}{N^2} F_C \cong 3.337152088 \times 10^{-4}\ newton. \tag{41}$$

The utmost important definition is: N^2 is the ratio of 'classical force limit' and 'nuclear weak force magnitude'. This is another significance of Avogadro number. Relation between nuclear strong force and weak force can be represented as

$$\sqrt{\frac{F_S}{F_W}} \cong 2\pi \ln\left(\frac{F_C}{F_W}\right) \cong 2\pi \ln\left(\frac{G_A}{G_C}\right) \cong 2\pi \ln\left(N^2\right). \tag{42}$$

where $F_S \cong 157.9944058$ *newton* can be called as the magnitude of the nuclear strong force. Characteristic nuclear size R_0 be expressed as

$$R_0 \cong \sqrt{\frac{e^2}{4\pi\epsilon_0 F_S}} \cong 1.208398568 \times 10^{-15}\ m. \tag{43}$$

Absolutely this can be considered as the space-time curvature at the nuclear charge surface provided there exists the atomic gravitational constant $N^2 G_C$. Now the proposed hypothetical dark matter characteristic mass unit can be represented as

$$m_X c^2 \cong \sqrt{\frac{e^2 F_W}{4\pi\epsilon_0}} \cong 1.731843735\ KeV. \tag{44}$$

Its existence has to be confirmed from experiments.

7.1 Mystery of proton and neutron rest masses

To a very great surprise it is noticed that,

$$m_p c^2 \cong \left(\frac{F_S}{F_W} + X_E^2 - \frac{1}{\alpha^2}\right) \sqrt{\frac{e^2 F_W}{4\pi\epsilon_0}} \cong 938.1791392\ MeV. \tag{45}$$

Here $m_p c^2$ is the rest energy of proton. This relation indicates that X_E^2 is a force ratio. It can be represented as

$$X_E^2 \cong \frac{4\pi\epsilon_0 G_A m_e^2}{e^2}. \tag{46}$$

where m_e is the electron rest mass. Thus X_E^2 may be referred to the gravitational and electromagnetic force ratio of electron where the operating

gravitational constant is N^2G_C but not G_C. Already $\frac{F_S}{F_W}$ is a force ratio. Hence one can say that $\frac{1}{\alpha^2}$ is also a force ratio or atleast it is related to a force ratio. Based on super symmetry in strong interaction Seshavatharam and Lakshminarayana [3, 25] suggested the same idea. From the above relations and equation (18) qualitatively and quantitatively α can be expressed as

$$\frac{1}{\alpha} \cong \frac{1}{2}\sqrt{X_E^2 - \frac{F_S}{4\pi^2 F_W}} \cong 136.9930484. \tag{47}$$

Neutron and proton mass difference can be expressed as

$$m_n c^2 - m_p c^2 \cong \sqrt{\frac{F_S}{F_W} + X_E^2}\ \sqrt{\frac{e^2 F_W}{4\pi\epsilon_0}} \cong 1.29657348\ MeV. \tag{48}$$

7.2 Magnetic moments of electron and nucleons

Interesting idea is that $\sin\theta_W$ can be considered as the ratio of up quark mass and down quark mass. Authors [25] suggested and implemented this idea in particle physics. It can be expressed as

$$\sin\theta_W \cong \frac{Up\ quark\ mass}{Down\ quark\ mass} \cong \frac{1}{\alpha X_E} \cong 0.464433353. \tag{49}$$

Paul Dirac [37] proposed his famous quantum theory of electron in 1928. With reference to the proposed nuclear weak force, magnetic moment of electron can be expressed as

$$\mu \cong \frac{ec}{2}\sqrt{\frac{e^2}{4\pi\epsilon_0 F_W}}\sin\theta_W. \tag{50}$$

Seshavatharam and Lakshminarayana [38] proposed this idea recently. Here $\sin\theta_W$ is the weak coupling angle. Y.K. Gambhir et al [39], N. Kaiser [40], Xiang-Song Chen et al [41] and V. Dimitrsinovic et al [42] discussed about the nucleon magnetic moments in terms of strong interaction. Similarly with reference to the proposed strong nuclear force, magnetic moment of nucleon can be expressed as

$$\mu \cong \frac{ec}{2}\sqrt{\frac{e^2}{4\pi\epsilon_0 F_S}}\sin\theta_W. \tag{51}$$

With reference to the characteristic nuclear radius or proton and neutron radii this relation can be expressed as

$$\mu \cong \frac{ecR_0}{2} \sin\theta_W. \tag{52}$$

Interesting observation is that neutron magnetic moment is matching at 0.866 fermi and proton magnetic moment is matching at 1.265 fermi.

7.3 The electroweak energy scale

Electron rest energy can be represented as

$$m_e c^2 \cong 2\sin^2\theta_W \times \sqrt{\frac{e^2 F_S}{4\pi\epsilon_0}} \cong 0.514\ MeV. \tag{53}$$

With 96.417% accuracy Fermi's weak coupling constant G_F can be expressed as

$$G_F \cong \frac{\sin^2\theta_W}{\sqrt{2}} \times \frac{F_W}{F_S} \times \hbar c \times \frac{e^2}{4\pi\epsilon_0 F_S} \cong \frac{\sin^2\theta_W}{\sqrt{2}} \times \frac{F_W}{F_S} \times \hbar c R_0^2. \tag{54}$$

Chris Quigg [43] discussed about the estimation of the weak coupling constant. Recommended value of $G_F \cong 1.435841042 \times 10^{-62}\ J.m^3$ and $\frac{G_F}{\hbar^3 c^3} \cong 1.166371 \times 10^{-5}\ GeV^{-2}$. Its obtained value is $G_F \cong 1.487247627 \times 10^{-62}\ J.m^3$ and $\frac{G_F}{\hbar^3 c^3} \cong 1.208129905 \times 10^{-5}\ GeV^{-2}$. Thus it is noticed that,

$$E_W \cong \sqrt{\frac{\hbar^3 c^3}{\sqrt{2} G_F}} \cong \frac{F_S}{F_W} \times m_e c^2 \cong 241.9277486\ GeV. \tag{55}$$

where E_W is the electroweak energy scale. One can say that, ratio of electroweak energy scale and rest energy of electron is close to the proposed strong and weak force ratio. This observation can be given a chance in understanding the 4 fundamental interactions.

7.4 The strong coupling constant

If E_a = asymmetry energy constant, E_c = coulombic energy constant of the semi empirical mass formula and α_s = strong coupling constant it is noticed that,

$$F_S \cong \frac{e^2}{4\pi\epsilon_0 R_0^2} \cong e^{\frac{1}{\alpha_s}} \times \frac{2E_a}{E_c} \times F_W. \tag{56}$$

Note that $\frac{E_c}{2E_a}$ plays a crucial role in nuclear stability. Claudia Glasman [44], J. Erler and P. Langacker [45] discussed about the estimation of the strong coupling constant. It can be fitted as

$$\frac{1}{\alpha_s} \cong \ln\left(\frac{E_c}{2E_a} \times \frac{F_S}{F_W}\right). \tag{57}$$

7.5 Electron in the Hydrogen atom and in the β - decay

In hydrogen atom, force of attraction between proton and electron can be represented as,

$$\frac{e^2}{4\pi\epsilon_0 a_0^2} \cong \left(\frac{E_c}{2E_a}\right)^2 F_W \tag{58}$$

Here a_0 is the Bohr radius. It can be expressed as

$$a_0 \cong \frac{2E_a}{E_c}\sqrt{\frac{e^2}{4\pi\epsilon_0 F_W}}. \tag{59}$$

Hence potential energy of electron in hydrogen atom can be given as

$$\frac{e^2}{4\pi\epsilon_0 a_0} \cong \left(\frac{E_c}{2E_a}\right)\sqrt{\frac{e^2 F_W}{4\pi\epsilon_0}} \cong \alpha^2 m_e c^2. \tag{60}$$

$$\alpha^2 \cong \left(\frac{E_c}{2E_a}\right)\sqrt{\frac{e^2}{4\pi\epsilon_0 G_A m_e^2}}. \tag{61}$$

Giving importance to the phenomena of β -decay, rest mass-energy of electron can be expressed as

$$m_e c^2 \cong \frac{1}{\alpha^2} \times \frac{E_c}{2E_a} \times \sqrt{\frac{e^2 F_W}{4\pi\epsilon_0}} \cong X_E \times \sqrt{\frac{e^2 F_W}{4\pi\epsilon_0}}. \tag{62}$$

Thus it is noticed that

$$\frac{E_c}{2E_a} \cong X_E \alpha^2. \tag{63}$$

Hence strong coupling constant can be obtained as

$$\frac{1}{\alpha_s} \cong \ln\left(X_E \alpha^2 \times \frac{F_S}{F_W}\right) \cong 8.914475771. \tag{64}$$

That means there is 'something' in this proposed 'atomic gravitational constant' and that secret has to be find out. The semi empirical mass formula energy coefficients can be obtained in this grand unification program. Nucleons rest masses can be co-related. Authors humble opinion is: in atomic system there exists a gravitational constant whose magnitude is N^2 times the classical gravitational constant. Even though this is unbelievable it plays a vital role in the unification of 'GEWS' interactions. This can be considered as the beginning of Avogadro's gravity for nuclear interactions. Seshavatharam and Laksminarayana [27] proposed new ideas in this connection.

8 Semi empirical mass formula energy constants

In this section authors made an attempt to couple the famous semi empirical mass formula with the grand unification scheme. In this scheme the new number $X_E \cong 295.0606338$ plays a very interesting role. Let E_a = asymmetry energy constant, E_c = coulomb energy constant, E_p = pairing energy constant, E_v = volume energy constant and E_s = surface energy constant. X_E be defined as follows.

$$X_E \cong \sqrt{\frac{4\pi\epsilon_0 G_A m_e^2}{e^2}} \cong 295.0606338. \tag{65}$$

where m_e is the rest mass of electron. W. D. Myers et al [46] estimated the atomic masses with various energy constants. With reference to the existing nuclear binding energy constants - empirically it can be suggested that

Z	A	Obtained Be, MeV
26	56	491.3
44	100	863.8
50	117	997.3
79	197	1555.3
92	238	1804.1

Table 2: Fitting of nuclear binding energy with proposed energy constants.

$$E_p \cong 2X_E\sqrt{\frac{\hbar c^5}{G_A}} \cong 11.96374935\ MeV. \tag{66}$$

$$E_a \cong 2E_p \cong 23.92749869\ MeV. \tag{67}$$

$$\sqrt{\frac{E_a}{E_c}+1} \cong \ln(X_E) \quad \text{and} \quad E_c \cong 0.763383059\ MeV. \tag{68}$$

$$E_a - E_v \cong E_s - E_p \cong 2\ln\left(\frac{X_E}{2}\right)E_c \cong 7.624721443\ MeV. \tag{69}$$

$$E_a + E_p \cong E_v + E_s \cong 3E_p \cong 35.89124805\ MeV. \tag{70}$$

Hence $E_v \cong 16.30277725\ MeV$ and $E_s \cong 19.58847079\ MeV$.

$$\frac{E_c}{2E_a} \cong 0.015952 \cong X_E\alpha^2 \cong 0.015712378. \tag{71}$$

See table-2 for nuclear binding energy. The existing nucleon-proton stability relation can be expressed as

$$Z_S \cong \frac{A}{2+0.0157A^{\frac{2}{3}}} \cong \frac{A}{2+\left(X_E\alpha^2\right)A^{\frac{2}{3}}}. \tag{72}$$

Here A is the mass number and Z_s is the stable isotope's proton number.

8.1 The nuclear stability factor

From the above obtained binding energy constants nuclear stability factor can be defined as

$$S_f \cong \frac{E_a}{E_c}\sqrt{\frac{E_s}{E_c}} \cong 158.7756104. \tag{73}$$

Empirically proton-neutron stability relation can be expressed as

$$A_S \cong 2Z + \frac{Z^2}{S_f} \cong 2Z + \frac{Z^2}{158.776}. \tag{74}$$

Here Z is the proton number and A_S is the stable mass number of Z. Roy Chowdhury et al [47] proposed a similar relation. For example, if Z= 29, A_S= 63.30; Z=47, A_S = 107.91; Z= 83, A_S= 209.39 and Z = 92, A_S = 237.30; By considering A as the fundamental input its corresponding stable Z = Z_S can be obtained as

$$Z_S \cong \left[\sqrt{\frac{A}{S_f} + 1} - 1\right] S_f. \tag{75}$$

Surprisingly it is noticed that this number S_f plays a crucial role in fitting the nucleons rest mass. Interesting observation is that

$$(m_n - m_p)\, c^2 \cong \ln\left(\sqrt{S_f}\right) m_e c^2. \tag{76}$$

Here m_n, m_p and m_e are the rest masses of neutron, proton and electron respectively.

8.2 The strong coupling constant and the nuclear stability factor

Semi empirically inverse of the strong coupling constant can be expressed as

$$\frac{1}{\alpha_s} \cong \ln\left(X_E^2\sqrt{\alpha}\right) \cong 8.914239916 \cong \frac{1}{0.112180063}. \tag{77}$$

This is a very interesting definition. Now the nuclear stability factor can be defined as

$$S_f \cong \frac{2}{\alpha_s^2} \cong 158.9273465. \tag{78}$$

Conclusion

Authors showed many applications of the existence of the atomic gravitational constant. Its existence as a 'true grand unified nuclear physical constant' can be confirmed. Reality can be understood only with interest, involvement and logical analysis. Now time has come to measure the value of the gravitational constant in atomic and nuclear space time curvature. Observing 'dark matter' is a very interesting and important job in cosmology. For any particle physicist it is very important to know its massive origin. In this paper a simple idea is proposed for understanding the massive origin of dark matter. For any new idea, for any physicist the final step is: 'to see/feel it experimentally' or 'to observe its direct and indirect implications'. Authors are working in this direction also. Note that human beings are part of this universal gravity. There are some natural restrictions to experiments. In a grand unified program 'absolute findings' can be understood but 'absolute measurements' can not be made by human beings. Authors request the whole science community to kindly look into this new approach.

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GENERALIZED ELECTRODYNAMICS: THE ORIGIN OF MASS FROM VACUUM

Sushama V. Vaidya
Physics Department
S. B. College of Science
Aurangabad-431001, India
sushansh@yahoo.com
M.T. Teli
Physics Department
Dr. Babasaheb Ambedkar
Marathwada University
Aurangabad-431004, India

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Abstract

Duality symmetrization of generalized Maxwell equations is done to establish generalized Dirac-Maxwell equations in electrodynamics and tensor forms of them are obtained suitable for getting Power and Force laws. The generalized Poynting four-vector is seen to satisfy relativistic energy-momentum relation giving rise to mass of particles in terms of fields which indicates that energy is carried by particles and not by fields. Two Lorentz invariant masses - scalar and pseudo-scalar- are seen to occur to form a complex mass which gives a binary mass compound. The model is tested for neutron and proton formation from bare nucleons and pions.

Key words: Generalized Dirac-Maxwell equations, Power and Force- law, Generalized Poynting vector, Mass from vacuum, Binary mass compound.

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1 Introduction

Maxwell's classical theory of electrodynamics with Lorenz condition is supposed to be a complete theory. Lorenz condition is needed there for obtaining inhomogeneous wave equations for potentials and making the theory gauge invariant. It implies charge conservation and also suggests that the potentials are unphysical. A good review literature on Lorenz condition is available ([4],[9],[10]). Heras [2] and Heras and Guillermo [3] have however, shown that potentials in the Lorenz gauge can be considered as physical because they separately satisfy causality and propagate with the speed of light and also because they can be written in covariant form.

If care is taken to maintain charge conservation and gauge invariance then one is free to write a theory which is to give inhomogeneous wave equations for potentials without the interference of Lorenz conditions. Such a theory has been formulated by Teli [13,14] in 1985 by generalizing Dirac- Maxwell equations after replacing Lorenz conditions by electric and magnetic scalar fields and attaching them to the electromagnetic three component field vectors. In this formalism the four vector potentials satisfy inhomogeneous wave equations without needing Lorenz condition. Such potentials produce electromagnetic field vectors identical to Maxwell's fields and also produce scalar fields arising from their non-vanishing four divergences. There are in all eight components of electromagnetic fields which obey equations similar to Dirac-Maxwell equations including monopoles. These generalized Dirac-Maxwell equations (GDM) are also expressed into tensor as well as quaternion forms. Generalized Poynting vector obtained from these equations includes a new vector orthogonal to the old Poynting vector. Additional pseudo-vector similar to Poynting vector is also obtained. The equations are duality invariant and also invariant under mass-less gauge. Teli and Jadhav [15] have also studied weak interactions by obtaining fields from the potentials given in Panofsky and Phillips [8] and there they showed that scalar fields occur if the charges are time dependent and also that time decaying charges introduce range to the force, the range being infinite when the charges are time independent.

Our Generalized Dirac-Maxwell equations are also well expressible in the powerful formalism in terms of Dirac bi-vectors based on Clifford algebra as developed by Rosa et al. [12], Rodrigues et al. [11] and Maia et al. [7]. This formalism gives in a natural way, in addition to string-free monopoles, the occurrence of scalar electromagnetic fields as Dirac bi-scalar.

Vlaenderen and Waser [18] have also introduced a scalar function in place of Lorenz condition and modified Maxwell's equations. Vlaenderen [17] also

obtained generalized Poynting theorem and showed that the generalized Poynting vector has component orthogonal to the old Poynting vector.

Lehnert [5] and Anastasovski et al. [1] have similarly replaced Lorenz condition by electric scalar function. They used it to define vacuum current density from its four-gradient. Lehnert [6] has also shown that the generalized Poynting vector has component vectors both in the axial as well as in the transverse direction of propagation. These workers have made detailed study of vacuum.

The purpose of this paper is to obtain generalized power and force laws from the GDM equations and to show that the fields create scalar and pseudo-scalar masses in vacuum and also that the pseudo-scalar mass requires the co-existence of both the electric scalar and magnetic pseudo-scalar fields otherwise it vanishes.

In the sec. (2) we symmetrize Lehnert and Vlaenderen's generalized Maxwell equations under full duality transformations. This re-establishes our GDM equations. In the sec. (3) we express the GDM-equations in tensor forms and obtain power and force laws in the sec. (4). Occurrence of masses from the fields in vacuum is investigated in the sec. (5). In the sec. (6) we test the theory with neutrons and protons which are the known binary mass particles. Important results are summarized and discussed in the last sec. (7).

2 Symmetrization of Generalized Maxwell Equations under Duality

In the notations of Teli [13] generalized Maxwell equations of Vlaenderen and Waser [18] and Lehnert [5] can be written as (bold faced letters indicate three- vectors everywhere)

$$\times \boldsymbol{H} - \partial_0 \boldsymbol{E} - {}' E_0 = \boldsymbol{j}^e, \qquad \boldsymbol{E} + \partial_0 E_0 = \rho^e$$

$$\times \boldsymbol{E} + \partial_0 \boldsymbol{H} = 0, \qquad \boldsymbol{H} = 0 \tag{1}$$

Their potential solutions are

$$\boldsymbol{H} = \times \boldsymbol{A}^e, \quad \boldsymbol{E} = -\partial_0 \boldsymbol{A}^e - {}' \varphi^e, \quad E_0 = \boldsymbol{A}^e + \partial_0 \varphi^e \tag{2}$$

Since Maxwell's equations in vacuum are invariant under duality transformations $\boldsymbol{H} \quad \boldsymbol{E}, \boldsymbol{E} \quad -\boldsymbol{H}$ eqns. (1) and (2) should also be so, i.e. they should be invariant under the extended duality transformations $\boldsymbol{H} \quad \boldsymbol{E}, \boldsymbol{E} \quad -\boldsymbol{H}, E_0 \quad -H_0, H_0 \quad E_0$ where for this purpose, we need to introduce

magnetic scalar field H_0. We extend these transformations further to include the sources and the potentials. We define the sets $\{e\}$ and $\{m\}$ as

$$\{e\} = \{\boldsymbol{E}, E_0, \boldsymbol{A}^e, \varphi^e, \boldsymbol{j}^e, \rho^e\}, \quad \{m\} = \{\boldsymbol{H}, H_0, \boldsymbol{A}^m, \varphi^m, \boldsymbol{j}^m, \rho^m\} \tag{3}$$

The *full duality transformations* are then defined as $\{m\}$ $\{e\}, \{e\}$ $-\{m\}$. The invariance of eqns. (1) and (2) under these full duality transformations requires the introduction of the corresponding magnetic quantities which are absent in them. The complete set of equations invariant under these transformations then becomes

$$\times \boldsymbol{H} - \partial_0 \boldsymbol{E} - \ E_0 = \boldsymbol{j}^e, \qquad \times \boldsymbol{E} + \partial_0 \boldsymbol{H} + \ H_0 = -\boldsymbol{j}^m \tag{4}$$

$$\boldsymbol{H} + \partial_0 H_0 = \rho^m, \qquad \boldsymbol{E} + \partial_0 E_0 = \rho^e \tag{5}$$

where

$$\boldsymbol{H} = \ \times \boldsymbol{A}^e - \partial_0 \boldsymbol{A}^m - \ \varphi^m, \quad \boldsymbol{E} = - \ \times \boldsymbol{A}^m - \partial_0 \boldsymbol{A}^e - \ \varphi^e \tag{6}$$

$$H_0 = \nabla \ \boldsymbol{A}^m + \partial_0 \varphi^m, \qquad E_0 = \nabla \ \boldsymbol{A}^e + \partial_0 \varphi^e \tag{7}$$

Eqns. (4) to (7) are the generalized Dirac-Maxwell (GDM) equations of Teli [13]. They now form the complete symmetrical theory of electrodynamics. Each of the eqns. (4) to (7) is invariant under the full duality transformations.

3 All in One: The Tensor form

It is convenient to write eqns. (4) and (5) and also (6) and (7) into single equations i.e. in tensor forms. Using covariant and contra-variant indices (Greek indices take values 0, 1, 2, 3 and italic indices 1, 2, 3) and the fundamental tensor $g^{\mu\nu}$ of signature (+, -, -, -) along with Einstein's summation convention, we write eqns. (4) to (7) into tensor forms as follows:

By defining complex four component quantities as

$$j^\mu = j^{m\mu} + ij^{e\mu}, \quad V^\mu = A^{m\mu} + iA^{e\mu}, \quad f^\mu = H^\mu + iE^\mu \tag{8}$$

we express eqns. (4) to (7) as

$$\boldsymbol{j} = \mathrm{i}\ \times \boldsymbol{f} - \partial_0 \boldsymbol{f} - \ f_0\ , \quad j_0 = \quad \boldsymbol{f} + \partial_0 f_0 \tag{9}$$

where

$$\boldsymbol{f} = -\mathrm{i}\ \times \boldsymbol{V} - \partial_0 \boldsymbol{V} - \ V_0\ , \ f_0 = \quad \boldsymbol{V} + \partial_0 V_0 \tag{10}$$

Eqns. (9) and (10) can be converted into tensor forms in *two* ways:

i) By treating j^μ and V^μ quantities as four-vectors while f^μ quantities as components of tensor:

In this case, eqns. (9) and (10) miniaturize as

$$\partial_\nu G^{\mu\nu} = j^\mu\ , \qquad G^{\mu\nu} = d^{\mu\nu\lambda\rho}\partial_\lambda V_\rho \tag{11}$$

where

$$G^{\mu\nu} = f_0 g^{\mu\nu} + f^{\mu\nu}\ , \ f^{\nu\mu} = -f^{\mu\nu}, \ f^{0i} = f^i, f^{ij} = if^k \tag{11a}$$

Here we have introduced the *fundamental hyper tensor* of rank four [16] defined as

$$d^{\mu\nu\lambda\rho} = g^{\mu\nu\lambda\rho} - i\varepsilon^{\mu\nu\lambda\rho} \tag{12}$$

$$g^{\mu\nu\lambda\rho} = g^{\mu\nu}g^{\lambda\rho} - g^{\mu\lambda}g^{\nu\rho} + g^{\mu\rho}g^{\nu\lambda} \tag{13}$$

$\varepsilon^{\mu\nu\lambda\rho}$ is the totally antisymmetric Levi-Civita symbol with $\varepsilon^{0123} = -1$.

ii) By treating f^μ quantities as four-vectors while taking j^μ and V^μ quantities as components of tensor:

For this case we have the representations

$$j^{\mu\nu} = d^{\ \mu\nu\lambda\rho}\partial_\lambda f_\rho\ , \qquad \partial_\nu V^{\mu\nu} = f^\mu \tag{14}$$

where

$$j^{\mu\nu} = j_0 g^{\mu\nu} + J^{\mu\nu}, \; J^{\nu\mu} = -J^{\mu\nu}, \; J^{0i} = j^i, J^{ij} = -ij^k \tag{15}$$

$V^{\mu\nu}$ has structure identical to $j^{\mu\nu}$. Divergences of eqns. (11) and (14) give the inhomogeneous equations for V^μ and f^μ fields:

$$\partial^2 V^\mu = j^\mu, \; \partial^2 f^\mu = \partial_\nu j^{\mu\nu} \tag{16}$$

The two descriptions given in eqns. (11) and (14) are equivalent because both of them represent the same set of eqns. (4) to (7). We can select the convenient form out of them for investigation of the physical quantities.

4 Power and Force Laws

For obtaining Power and Force laws which are also known as Poynting theorems, the equation useful for this purpose is eqn. (14) viz.

$$d^{\mu\nu\lambda\rho}\partial_\lambda f_\rho = j^{\mu\nu} \tag{17}$$

This is a linear inhomogeneous differential equation in f similar in form to Dirac's quantum mechanical equation for electron. As such equation of continuity should follow from it. We follow the same procedure as used in quantum mechanics. Complex conjugate of eqn. (17) gives

$$d^{\mu\nu\lambda\rho}\partial_\lambda f^{*}_{\;\rho} = j^{*\mu\nu} \tag{17a}$$

Taking scalar product of (17) with $f^{*}_{\;\nu}$ and of (17a) with f_ν and adding together the results we get

$$\frac{1}{2}\left[d^{*\mu\nu\lambda\rho} f^{*}_{\;\nu}\partial_\lambda f_\rho + d^{\mu\nu\lambda\rho} f_\nu \partial_\lambda f^{*}_{\;\rho}\right] = \frac{1}{2}\left(j^{*\mu\nu} f_\nu + j^{\mu\nu} f^{*}_{\;\nu}\right) = F^\mu \tag{18}$$

where F^μ is the force exerted by the fields on the sources. Interchanging the indices ν and ρ in the second term on the *l.h.s.* of (18) we get

$$\frac{1}{2}\partial_\lambda\left[d^{\mu\nu\lambda\rho} f^{*}_{\;\nu} f_\rho\right] = \frac{1}{2}\left(j^{*\mu\nu} f_\nu + j^{\mu\nu} f^{*}_{\;\nu}\right) = F^\mu \tag{19}$$

This is the divergence of the tensor

$$T^{\mu\lambda} = \frac{1}{2} d^{\ \mu\nu\lambda\rho} f_{\ \nu} f_{\rho} \tag{20}$$

Eqn. (19) is the force law. For $\mu = 0$ it gives Poynting's theorem in generalized form (power law). If $F^{\mu} = 0$, eqn. (19) becomes the equation of continuity or conservation law for the tensor $T^{\mu\lambda}$.

Similarly, if we consider the interaction between the source and its own field (*self interaction*) the force experienced by the source from its own field will be given by

$$d^{\ \mu\nu\lambda\rho} f_{\nu} \partial_{\lambda} f_{\rho} = j^{\mu\nu} f_{\nu} = F^{\mu}_{self} \tag{21}$$

This can be written as

$$\frac{1}{2}\{[d^{\ \mu\nu\lambda\rho} f_{\nu} \partial_{\lambda} f_{\rho}] + [d^{*\mu\nu\lambda\rho} f_{\nu} \partial_{\lambda} f_{\rho}]\} = F^{\mu}_{self} \tag{22}$$

This simplifies to

$$F^{\mu}_{self} = \frac{1}{2}\{\partial_{\lambda}[g^{\mu\nu\lambda\rho} f_{\nu} f_{\rho}] + i\varepsilon^{\mu\nu\lambda\rho}[(f_{\nu} \partial_{\lambda} f_{\rho} - f_{\rho} \partial_{\lambda} f_{\nu})\} \tag{23}$$

The second term in (23) contributes to self force so that we get the effective force law as

$$F^{\mu}_{eff} = F^{\mu}_{self} - \frac{1}{2} i\varepsilon^{\mu\nu\lambda\rho}(f_{\nu} \partial_{\lambda} f_{\rho} - f_{\rho} \partial_{\lambda} f_{\nu}) = \frac{1}{2} \partial_{\lambda}[g^{\mu\nu\lambda\rho} f_{\nu} f_{\rho}] \tag{24}$$

This is the divergence of the self energy- momentum tensor given by

$$T^{\mu\lambda}_{self} = \frac{1}{2}[g^{\mu\nu\lambda\rho} f_{\nu} f_{\rho}] \tag{25}$$

and gives the *self-force law*.

5 Massive Vacuum: The origin of Mass from the Fields

In vacuum $j^{\mu\nu}$ or j^{μ} are zero and the tensor $T^{\mu\lambda}$ given by eqn. (20) is conserved. Using equations (12) and (13) into eqn. (20) we get

$$T^{\mu\lambda} = \frac{1}{2}\left[\left(f^{\ \mu}f^{\lambda} + f^{\mu}f^{\ \lambda}\right) - g^{\mu\lambda}f^{\ \nu}f_{\nu} + i\varepsilon^{\mu\nu\lambda\rho}f_{\ \nu}f_{\rho}\right] \tag{26}$$

Using eqn. (8) into this equation, we obtain

$$T^{\mu\lambda} = H^{\mu}H^{\lambda} + E^{\mu}E^{\lambda} - \frac{1}{2}g^{\mu\lambda}\left(H^{\nu}H_{\nu} + E^{\nu}E_{\nu}\right) + \varepsilon^{\mu\nu\lambda\rho}E_{\nu}H_{\rho} \tag{27}$$

The four sectors of this tensor are given as

$$T^{00} = S_0 = \frac{1}{2}\left(H_0^2 + E_0^2 + \boldsymbol{H}^2 + \boldsymbol{E}^2\right) \tag{28}$$

$$\left(T^{0i}\right) = \boldsymbol{S} = H_0\boldsymbol{H} + E_0\boldsymbol{E} + \boldsymbol{E} \times \boldsymbol{H} \tag{29}$$

$$\left(T^{i0}\right) = H_0\boldsymbol{H} + E_0\boldsymbol{E} - \boldsymbol{E} \times \boldsymbol{H} \tag{30}$$

$$T^{ij} = \left(H^iH^j + E^iE^j\right) - \frac{1}{2}g^{ij}\left(H^{\rho}H_{\rho} + E^{\rho}E_{\rho}\right) + \varepsilon^{i\nu j\rho}E_{\nu}H_{\rho} \tag{31}$$

Eqn. (27) is a smooth generalization of Maxwell's energy-momentum-stress tensor. It is symmetric except in the last term. Eqn. (28) gives the energy density while (29) gives the generalized Poynting vector. Eqn. (30) gives field momentum density while (31) gives the stress tensor or moment of momentum. The contributions of the scalar fields H_0 and E_0 to these quantities are obvious and self explanatory. The vectors $\boldsymbol{E} \times \boldsymbol{H}$ and $\boldsymbol{H}H_0 + \boldsymbol{E}E_0$ are mutually orthogonal. We notice that

$$S_{\mu}S^{\mu} = S_0^2 - \boldsymbol{S}^2 = m_1^2 + m_2^2 \tag{32}$$

where

$$m_1^2 = \frac{1}{4}\left[\left(E_0^2 + \boldsymbol{H}^2\right) - \left(H_0^2 + \boldsymbol{E}^2\right)\right]^2 \tag{33}$$

and

$$m_2^2 = \left(H_0E_0 - \boldsymbol{H}\ \ \boldsymbol{E}\right)^2 \tag{34}$$

Eqn. (32) is the relativistic energy-momentum relation and so the *r.h.s.* of it is the square of Lorentz invariant mass. It is however composed of two

masses m_1 and m_2. Because of this mass of the radiation, the field energy does not propagate with the speed of light.

Similarly in vacuum, the second term in (23) vanishes because f_ν are plane waves (eqn. (16)) and so F^{μ}_{eff} vanishes and the tensor given in (25) gets conserved. Using eqn. (13), eqn. (25) expands to

$$T^{\mu\lambda}_{self} = f^{\mu} f^{\lambda} - \frac{1}{2} g^{\mu\lambda} (f^{\nu} f_{\nu}) \tag{35}$$

This tensor is symmetric. The T^{00}_{self} and T^{01}_{self} components are given as

$$T^{00}_{self} = S_{10} = \frac{1}{2}[{f_0}^2 + \boldsymbol{f}^2] \tag{36}$$

$$T^{01}_{self} = \boldsymbol{S}_1 = f_0 \boldsymbol{f} \tag{37}$$

These are the complex energy and momentum and hence not directly observable. They satisfy energy-momentum relation

$$\frac{1}{4}({f_0}^2 + \boldsymbol{f}^2)^2 - {f_0}^2 \boldsymbol{f}^2 = \frac{1}{4}({f_0}^2 - \boldsymbol{f}^2)^2 = (m_1 + i m_2)^2 \tag{38}$$

The *r.h.s.* is the square of a complex mass $m = m_1 + i m_2$ indicating that the object formed by the fields acquires complex mass. It is composed of two internal particles having component masses given by eqns. (33) and (34). The observable mass is given

$$\left|\frac{1}{2}[{f_0}^2 - \boldsymbol{f}^2]\right|^2 = |m_1 + i m_2|^2 = m_1^2 + m_2^2 = |m|^2 \tag{39}$$

This indicates that the *r.h.s.* of eqn. (32) is $|m|^2$. The interpretation of these results can be that fields compose a complex particle having complex energy, momentum and mass as given by eqns. (36), (37) and (38). Such particle is in virtual bound state before emission. When it is radiated out in free space it is released with real energy and momentum given by eqns.(28) and (29) and observable mass as given by the connecting eqns. (32), (33), (34) and (39).

Now we have seen that vacuum emits massive compound mass radiation which gets energy and momentum from the fields and carries them along with it. However according to eqn. (16) the fields are plane waves in vacuum

propagating with the speed of light. Thus field waves propagate with the speed of light but field energy does not because the object formed by the fields acquires mass and so it travels with speed less than the speed of light. The field waves, therefore, cannot carry energy along with them. It is carried along with the particle radiation. There are now two velocities involved in the theory. One is the wave velocity with which the wave propagates with the speed of light but does not carry energy with it. The second one is the velocity with which the particle radiation formed from the fields travels and carries energy with it. In Maxwellian electrodynamics the electromagnetic waves carry along with them energy with the speed of light. This is so because in this case wave velocity and photon-particle radiation velocity are equal. The inclusion of the scalar fields in electrodynamics however, makes the particle velocity different from the field wave velocity. Wave velocity and particle velocity can be made the same by redefining vacuum which we do not consider here.

Now since, fields in vacuum are plane waves propagating with the speed of light we can see from eqns. (4) and (5) that $\boldsymbol{E}\ \boldsymbol{H} = 0$ if either $E_0 = 0$ or $H_0 = 0$. But $\boldsymbol{E}\ \boldsymbol{H}$ does not vanish if both E_0 and H_0 are non-zero. This means according to eqn. (34) that for the existence of m_2, both E_0 and H_0 must coexist such that $H_0 E_0 \neq \boldsymbol{H}\ \boldsymbol{E}$ otherwise m_2 vanishes.

6 Evidence for Compound Mass

In nuclear and Particle Physics compound mass formation is common in particle reactions. However, it is not known whether the mass components are related by the formula given in eqn. (32) or (39). We investigate here the case of simple and well established particles nucleons which are known binary compounds of bare nucleon and pions. Since the masses m_1 and m_2 arising from the fields are Lorentz invariants, it does not matter what kind of Lorentz group transforms the fields which give rise to the masses. We assume that nuclear fields also produce masses which obey eqn. (39). Let m_1 belong to bare nucleon and m_2 being pseudo-scalar belongs to pion. Let the nucleon capture neutral pion π^0 forming a stable proton. Then the mass formula (39) gives

$$M_p^2 = m_1^2(p) + m_{\pi 0}^2 \tag{40}$$

Similarly we write for neutron mass

$$M_n^2 = m_1^2(n) + m_{\pi-}^2 \tag{41}$$

The mass m_1 of bare nucleon is the same for bare neutrons and bare protons because in eqn. (33) the strengths of the fields are the same for protons and neutrons in view of charge symmetry and charge independence character of nuclear force. However, according to eqn. (34) pseudo-scalar mass can be varied for pions by changing the angle between the $\boldsymbol{H}$ and the $\boldsymbol{E}$ field vectors.

Now standard data gives

$$M_p = 938.45 \text{ MeV}, M_n = 939.85 \text{ MeV}, m_{\pi 0} = 135 \text{ MeV}, m_{\pi-} = 141.5 \text{ MeV}.$$

With this data eqn. (40) gives m_1 (p) = 928.7 MeV while eqn. (41) gives m_1 (n) = 929.1 MeV. The two estimates of the bare nucleon mass m_1 excellently agree with each other as expected because they should give $m_1(n) = m_1(p)$. Their difference is only 0.4 MeV which can be attributed to the experimental uncertainty in the measurements of pion masses. This strongly supports the eqns. (32) and (39) and hence the theory. This also reveals that nucleons are candidates of our GDM- theory applicable to nuclear fields.

7 Discussion

The duality symmetrization of extended Maxwell equations of Vlaenderen ([17],[18]) and Lehnert [5] with the introduction of electric scalar field in place of Lorenz conditions has led us to the Generalized Dirac-Maxwell equations of Teli ([13],[14]. The covariant formulation of these equations in Tensor forms has enabled us to smoothly obtain generalized energy-momentum stress tensor by a standard method used in quantum mechanics. The interesting feature of the generalized Poynting four vector is that it satisfies relativistic energy-momentum relation giving rise to Lorentz invariant mass in terms of the fields. The square of this mass is the sum of the squares of the two component masses one of them being a scalar while the other a pseudo-scalar. Self interaction of the source with its own fields has revealed us that the mass of particle formed from the fields is complex, the real part of which is a scalar mass while the imaginary part of it is the pseudo-scalar mass. These consequences are tested with the known binary mass particles protons and neutrons which are found to fit with the theory without any ambiguity.

The overall impression is that the insertion of scalar fields in Maxwell's equations in place of Lorenz gauge leads to evolution of scalar and pseudo-scalar masses from the fields even in vacuum. This also clarifies that energy is carried by particles and not by the fields. It is also seen that simultaneous existence of electric and magnetic scalar fields is essential for the occurrence of pseudo-scalar mass for the formation of compound binary mass. Existence of pions is then the evidence for the occurrence of both the scalar and the pseudo-scalar fields. The whole theory can also be formulated in terms of Dirac bi-vectors using Clifford algebra as developed in refs. ([7],[11],[12]).

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ON THE MATHEMATICAL SENSE OF RENORMALIZATION

Alexander Stoyanovsky
Russia State University of Humanities
Miusskaya pl. 6
Moscow 125993, Russia
alexander.stoyanovsky@gmail.com

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Abstract

We place the renormalization procedure in quantum field theory into the familiar mathematical context of quantization of Poisson algebras. The Poisson algebra in question is the algebra of classical field theory Hamiltonians constructed in a previous paper [4]. Its quantum deformations presumably contain (non-canonically) the algebra of functional differential operators. We explain that this picture contains renormalization as a natural ingredient.

Keywords: Field theory; Hamiltonian; Schwartz space; Functional derivative; Poisson algebra; Quantization; Functional differential operator.

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Introduction

The aim of this paper is to give a mathematical background to the notion of renormalization in quantum field theory (QFT). Some interesting attempts to understand renormalization mathematically have been made many times, see, for example, [1–3] and references therein. However, none of them was considered as satisfactory. For example, the construction of Connes and Kreimer [1] depends on a particular choice of regularization scheme, etc. We are going to give a transparent mathematical explanation of renormalization.

The key point in our approach is the construction of the Poisson algebra of classical Hamiltonians in field theory [4]. We recall this construction and explain its naturalness in §1. In [4] also the problem has been posed to quantize this Poisson algebra. When restricted to the Poisson subalgebra of regular (non-local) Hamiltonians, this problem has a trivial solution: a quantization is the algebra of functional differential operators (or the infinite dimensional Moyal algebra, which is the same). However, when we consider also singular (local) Hamiltonians, the situation drastically changes. The problem of quantization (or deformation quantization) becomes difficult and reminds the Kontsevich approach to deformation quantization of Poisson manifolds [5]. This approach is closely related to QFT [6], so one can expect that solution of our quantization problem can be achieved by QFT methods. Anyway, provided that a quantization has been constructed, we can obtain a non-canonical embedding of the algebra of functional differential operators into the constructed quantum algebra. However, this embedding cannot agree with taking the (non-canonical) classical symbol of a quantum Hamiltonian. Therefore we obtain a (non-unique) map from the space of classical non-local Hamiltonians to itself. This map is the required renormalization map. Details are exposed in §2.

1 Construction of the Poisson algebra of classical Hamiltonians in field theory

The generally covariant Hamiltonian classical field theory is exposed in detail in [7] and references therein.

Let us pose the following question. Let $\varphi = \varphi(\mathbf{x})$ be the classical (scalar for simplicity) field on a space-like surface C in space-time $\mathbf{R}^{n+1}$, $\mathbf{x} =$

$(x_1, \ldots, x_n)$ be coordinates on the surface, and let $\pi = \pi(\mathbf{x})$ be the conjugate momentum. Both φ and π belong to the Schwartz space S of smooth functions rapidly decreasing at infinity. The question is: what functionals $H(\varphi, \pi)$ can be classical field theory Hamiltonians? This class of functionals should contain the known examples, and should agree with the picture for free field (quadratic Hamiltonians).

Recall that the usual known examples of Hamiltonians are integrals over C of polynomial densities depending locally on φ and π. On the other hand, in the case of free Klein–Gordon field, the evolution operators from one space-like surface to another belong to the group of continuous symplectic transformations of the symplectic topological vector space $S \oplus S$. Therefore it is natural to expect that the quadratic Hamiltonians form the Lie algebra of this group.

It is easy to see that the answer to our question should be the following.

Definition.[4] A continuous polynomial functional $H(\varphi, \pi)$ on $S \times S$ is called a Hamiltonian (or a symbol) if its first functional differential δH, which is a linear functional on test functions $(\delta\varphi, \delta\pi) \in S \oplus S$ for every (φ, π), belongs to $S \oplus S \subset S' \oplus S'$ (here S' is the space of tempered distributions dual to S). Moreover, δH should be infinitely differentiable as an $S \oplus S$-valued functional on $S \times S$.

Denote the space of symbols by *Symb*.

Proposition. *Symb* is a topological Poisson algebra with respect to the standard Poisson bracket

$$\{H_1, H_2\} = \int \left(\frac{\delta H_1}{\delta \pi(\mathbf{x})} \frac{\delta H_2}{\delta \varphi(\mathbf{x})} - \frac{\delta H_1}{\delta \varphi(\mathbf{x})} \frac{\delta H_2}{\delta \pi(\mathbf{x})} \right) d\mathbf{x}. \tag{1}$$

Proof is straightforward.

Any $H \in Symb$ has the form

$$H(\varphi, \pi) = \sum_{k=0}^{N} \sum_{l=0}^{M} H_{k,l}(\varphi, \pi), \tag{2}$$

$$H_{k,l}(\varphi, \pi) = \frac{1}{k!l!} \int a_{k,l}(\mathbf{x}_1, \ldots, \mathbf{x}_k; \mathbf{y}_1, \ldots, \mathbf{y}_l)$$
$$\times \varphi(\mathbf{x}_1) \ldots \varphi(\mathbf{x}_k) \pi(\mathbf{y}_1) \ldots \pi(\mathbf{y}_l) d\mathbf{x}_1 \ldots d\mathbf{x}_k d\mathbf{y}_1 \ldots d\mathbf{y}_l$$

for certain tempered distributions $a_{k,l}$ symmetric in $\mathbf{x}_1, \dots, \mathbf{x}_k$ and in $\mathbf{y}_1, \dots, \mathbf{y}_l$ (by the Schwartz kernel theorem).

Definition. If all $a_{k,l}$ are smooth functions rapidly decreasing at infinity then the Hamiltonian H is called *regular*. Otherwise it is called *singular*.

Denote the subspace of *Symb* consisting of regular Hamiltonians by

$$Symb^{\mathrm{reg}}.$$

Clearly, it is a Poisson subalgebra dense in *Symb*.

2 The quantization problem and renormalization

2.1

Definition. A *quantization* (*deformation quantization*) of the Poisson algebra *Symb* is a continuous associative product

$$(H_1, H_2) \to H_1 * H_2 \tag{3}$$

on the topological vector space *Symb* smoothly (resp. formally) depending on a parameter h such that

$$\begin{array}{ll} \text{i)} & H_1 * H_2 = H_1 H_2 + O(h), \\ \text{ii)} & [H_1, H_2] \stackrel{\text{def}}{=} H_1 * H_2 - H_2 * H_1 = ih\{H_1, H_2\} + O(h^2), \\ \text{iii)} & \text{if } H_1, H_2 \text{ are quadratic } (k+l \le 2 \text{ in (2)}) \text{ then} \\ & [H_1, H_2] = ih\{H_1, H_2\}. \end{array} \tag{4}$$

Two (deformation) quantizations $*_1$ and $*_2$ are called *equivalent* if there exists a linear map from *Symb* to itself smoothly (resp. formally) depending on h, of the form $Id + O(h)$, which takes $*_1$ to $*_2$.

Problems. Find a (deformation) quantization of *Symb*. Classify all (deformation) quantizations up to equivalence.

2.2 Discussion of the quantization problem and renormalization

Clearly, the Poisson subalgebra $Symb^{\mathrm{reg}} \subset Symb$ admits a quantization to the algebra of regular functional differential operators with the usual formula for the product of symbols,

$$H_1 *_{Diff} H_2(\varphi,\pi) = \exp\left(ih\int \frac{\delta}{\delta\pi_1(\mathbf{x})}\frac{\delta}{\delta\varphi_2(\mathbf{x})}d\mathbf{x}\right)$$

$$H_1(\varphi_1,\pi_1)H_2(\varphi_2,\pi_2)|_{\varphi_1=\varphi_2=\varphi,\pi_1=\pi_2=\pi},$$

or to the Moyal algebra

$$H_1 *_{Moyal} H_2(\varphi,\pi) = \exp\frac{ih}{2}\int\left(\frac{\delta}{\delta\pi_1(\mathbf{x})}\frac{\delta}{\delta\varphi_2(\mathbf{x})} - \frac{\delta}{\delta\pi_2(\mathbf{x})}\frac{\delta}{\delta\varphi_1(\mathbf{x})}\right)d\mathbf{x}$$

$$H_1(\varphi_1,\pi_1)H_2(\varphi_2,\pi_2)|_{\varphi_1=\varphi_2=\varphi,\pi_1=\pi_2=\pi}.$$

It is easy to see that these two quantizations are equivalent.

However, these quantizations cannot be extended to the whole *Symb*, because the products contain pairings of higher functional derivatives which are distributions rather than smooth functions. (Recall that in *Symb* only the first functional derivatives are functions.) Therefore, a product formula for *Symb* should be constructed in a completely different way, it should not be a bidifferential operator. We hope that such a formula can be found by QFT methods, cf. [6].

Assume that such a quantization $*$ has been found. Moreover, assume that we have an inclusion of quantum algebras

$$R : (Symb^{\mathrm{reg}}, *_{Diff}) \hookrightarrow (Symb, *).$$

Note that for $H \in Symb^{\mathrm{reg}}$, we have, in general, $R(H) \neq H$. Moreover, for a family H_Λ, $\Lambda \in \mathbf{R}$ of regular Hamiltonians which tend to a non-regular Hamiltonian H as $\Lambda \to \infty$, we have $R^{-1}(H_\Lambda) \to \infty$.

In Hamiltonian quantum field theory, the map $H_\Lambda \to R^{-1}(H_\Lambda)$ is called the *renormalization map*.

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STRONG NUCLEAR GRAVITY-A BRIEF REPORT

U. V. S. Seshavatharam
Honorary Faculty, I-SERVE
Alakapuri, Hyderabad-35, India
seshavatharam.uvs@gmail.com
Prof. S. Lakshminarayana
Department of Nuclear Physics
Andhra University
Visakhapatnam-03, India
lnsrirama@yahoo.com

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Abstract

Key conceptual link that connects the gravitational force and non-gravitational forces is - the classical force limit $\left(\frac{c^4}{G}\right)$. For mole number of particles, if strength of gravity is $(N.G)$, any one particle's weak force magnitude is $F_W \cong \frac{1}{N}\cdot\left(\frac{c^4}{N.G}\right) \cong \frac{c^4}{N^2G}$. Ratio of 'classical force limit' and 'weak force magnitude' is N^2. This can be considered as the beginning of 'strong nuclear gravity'. Assumed relation for strong force and weak force magnitudes is $\sqrt{\frac{F_S}{F_W}} \cong 2\pi \ln\left(N^2\right)$. If m_p is the rest mass of proton it is noticed that $\ln\sqrt{\frac{e^2}{4\pi\varepsilon_0 G m_P^2}} \cong \sqrt{\frac{m_p}{m_e} - \ln\left(N^2\right)}$. From SUSY point of view, 'integral charge quark fermion' and 'integral charge quark boson' mass ratio is $\Psi = 2.262218404$ but not unity. With these advanced concepts starting from nuclear stability to charged leptons, quarks, electroweak bosons and charged Higgs boson's origin can be understood. Finally an "alternative" to the 'standard model' can be developed.

Keywords: Avogadro number, classical gravitational constant, grand unification, strong nuclear gravity, charged lepton masses, nuclear stability, nucleon rest masses, super symmetry, quark physics and electroweak physics.

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1 Introduction

Unification means : finding the similarities, finding the limiting physical constants, finding the key numbers, coupling the key physical constants, coupling the key physical concepts, coupling the key physical properties, minimizing the number of dimensions, minimizing the number of inputs and implementing the key physical constant or key number in different branches of physics. This is a very lengthy process. In all these cases observations, interpretations, experiments and imagination play a key role. The main difficulty is with interpretations and observations. As the interpretation changes physical concept changes, physical equation changes and finally the destiny changes. Universe is a very big laboratory and its life span is very large. Modern physics is having only and hardly 200 years of strong scientific back ground. Strong motivation, good reasoning, nature friendly concepts, simplicity and applicability are the most favourable and widely accepted qualities of any new model ([1], [2], [3]).

Note that in the atomic or nuclear physics, till today no one measured the gravitational force of attraction between the proton and electron and experimentally no one measured the value of the gravitational constant. Physicists say, if strength of strong interaction is unity, with reference to the strong interaction, strength of gravitation is 10^{-39}. The fundamental question to be answered is: is mass an inherent property of any elementary particle? To unify 2 interactions if 5 dimensions are required, for unifying 4 interactions 10 dimensions are required. For 3+1 dimensions if there exists 4 (observed) interactions, for 10 dimensions there may exist 10 (observable) interactions. To unify 10 interactions 20 dimensions are required. From this idea it can be suggested that- with 'n' new dimensions 'unification' problem can not be resolved.

As the culmination of his life work, Einstein wished to see a unification of gravity and electromagnetism as aspects of one single force. In modern language he wished to unite electric charge with the gravitational charge (mass) into one single entity. Further, having shown that mass the 'gravitational charge' was connected with space-time curvature, he hoped that the electric charge would likewise be so connected with some other geometrical property of space-time structure. For Einstein the existence, the mass, the charge of the electron and the proton the only elementary particles recognized back in 1920s were arbitrary features. One of the main goals of a unified theory should explain the existence and calculate the properties of matter.

Stephen Hawking [4] - in his famous book - says: It would be very difficult to construct a complete unified theory of everything in the universe all at one go. So instead we have made progress by finding partial theories that describe a limited range of happenings and by neglecting other effects or approximating them by certain numbers. (Chemistry, for example, allows us to calculate the interactions of atoms, without knowing the internal structure of an atomic nucleus.) Ultimately, however, one would hope to find a complete, consistent, unified theory that would include all these partial theories as approximations, and that did not need to be adjusted to fit the facts by picking the values of certain arbitrary numbers in the theory. The quest for such a theory is known

as "the unification of physics". Einstein spent most of his later years unsuccessfully searching for a unified theory, but the time was not ripe: there were partial theories for gravity and the electromagnetic force, but very little was known about the nuclear forces. Moreover, Einstein refused to believe in the reality of quantum mechanics, despite the important role he had played in its development.

The first step in unification is to understand the origin of the rest mass of a charged elementary particle. Second step is to understand the combined effects of its electromagnetic (or charged) and gravitational interactions. Third step is to understand its behaviour with surroundings when it is created. Fourth step is to understand its behaviour with cosmic space-time or other particles. Right from its birth to death, in all these steps the underlying fact is that whether it is a strongly interacting particle or weakly interacting particle, it is having some rest mass. To understand the first 2 steps somehow one must implement the gravitational constant in sub atomic physics.

2 Gravitational constant and Avogadro number in unification

The accuracy of the measured value of G has increased only modestly since the original Cavendish experiment. G is quite difficult to measure, as gravity is much weaker than other fundamental forces, and an experimental apparatus cannot be separated from the gravitational influence of other bodies. Furthermore, gravity has no established relation to other fundamental forces, so it does not appear possible to calculate it indirectly from other constants that can be measured more accurately, as is done in some other areas of physics. In 2007 P.J. Mohr and B.N. Taylor [5] recommended a value of G = $6.6742867 \times 10^{-11} \mathrm{m^3 Kg^{-1} sec^{-2}}$. Based on the newly developed interferometry techniques discussed by B. Fixler et al [6], measured value of G = $6.693 \times 10^{-11} \mathrm{m^3 Kg^{-1} sec^{-2}}$. Fitting the gravitational constant with the atomic and nuclear physical constants is a challenging task.

The strong or atomic gravitational constant is supposed physical constant of strong gravitation, involved in the calculation of the gravitational attraction at the level of elementary particles and atoms. The idea of strong gravity originally referred specifically to mathematical approach of Abdus Salam of unification of gravity and quantum chromo-dynamics, but is now often used for any particle level gravity approach. In literature one can refer the works of Abdus Salam, C. Sivaram, Sabbata, A.H. Chamseddine, J. Strathdee, Usha Raut, K. P. Sinha, J.J.Perng, E. Recami, R. L. Oldershaw, K.Tennakone, S.I Fisenko and S.G.Fedosion ([7] -[11]).

The key conceptual link that connects the gravitational and non-gravitational forces is - the classical force limit $\left(\frac{c^4}{G}\right)$. Seshavatharam [12] discussed about the vital role of the classical force limit in Black hole and Planck scale physics. For mole number of particles, if strength of gravity is $(N.G)$, any one particle's

weak force magnitude is $F_W \cong \frac{1}{N} \cdot \left(\frac{c^4}{N.G}\right) \cong \frac{c^4}{N^2 G}$. Ratio of 'classical force limit' and 'weak force magnitude' is N^2. This can be considered as the beginning of 'strong nuclear gravity'. If $\left(\frac{c^4}{G}\right)$ is the 'limit of classical force', in a grand unified scheme $\frac{c^4}{N^2 G}$ can be defined as the 'characteristic weak force magnitude' and $\sqrt{\frac{e^2}{4\pi\varepsilon_0} \cdot \frac{c^4}{N^2 G}}$ can be defined as the 'characteristic weak energy constant'. This can be considered as the beginning of 'strong (nuclear) gravity' . Seshavatharam and Lakshminarayana [13 - 22] proposed interesting concepts in this new direction.

In his large number hypothesis P. A. M. Dirac [23, 24] compared the ratio of characteristic size of the universe and classical radius of electron with the electromagnetic and gravitational force ratio of electron and proton. If the cosmic closure density is, $\rho_0 \cong \frac{3H_0^2}{8\pi G}$, number of nucleons in a Euclidean sphere of radius $\left(\frac{c}{H_0}\right)$ is equal to $\frac{4\pi}{3m_n}\left(\frac{c}{H_0}\right)^3 \cdot \frac{3H_0^2}{8\pi G} \cong \frac{c^3}{2Gm_n H_0} \cong \frac{c}{H_0} \div \frac{2Gm_n}{c^2}$. It can be suggested that coincidence of large number ratios reflects an intrinsic property of nature.

It can be supposed that elementary particles construction is much more fundamental than the black hole's construction. If one wishes to unify electroweak, strong and gravitational interactions it is a must to implement the classical gravitational constant G in the sub atomic physics. By any reason if one implements the planck scale in elementary particle physics and nuclear physics automatically G comes into subatomic physics. Then a large 'arbitrary number' has to be considered as a proportionality constant. With this large arbitrary number it is be possible to understand the mystery of the strong interaction and strength of gravitation. The basic and important problem is : How to select the 'arbitrary number' ? For this purpose 'mole' concept can be considered as a fundamental tool. The combination of Avogadro number and the classical gravitational constant generates the 'effective' 'strong gravitational constant'. Semi empirically it is noticed that

$$\ln \sqrt{\frac{e^2}{4\pi\varepsilon_0 G m_P^2}} \cong \sqrt{\frac{m_p}{m_e} - \ln\left(N^2\right)} \tag{1}$$

where m_p is the proton rest mass and m_e is the electron rest mass. From this expression

$$G \cong \left(e^{\sqrt{\frac{m_p}{m_e} - \ln(N^2)}}\right)^{-2} \cdot \frac{e^2}{4\pi\varepsilon_0 m_P^2} \cong 6.666270179 \times 10^{-11}\ \mathrm{m^3 Kg^{-1} sec^{-2}} \tag{2}$$

Here the important question is: What is the role of squared Avogadro number in grand unified physics? In the foregoing sections it is discussed in depth with many interesting results.

3 To fit the gravitational constant with atomic constants

It is well established that, in β decay, neutron emits an electron and transforms to proton. Thus the nuclear charge changes and the nucleus gets stability. From the semi empirical mass formula, it is established that,

$$Z \cong \frac{A}{2 + (E_c/2E_a)\, A^{2/3}}. \tag{3}$$

where Z = number of protons of the stable nucleus and A =number of nucleons in the stable nucleus. E_a and E_c are the asymmetry and coulombic energy constants. Semi empirically it is noticed that,

$$A_S \cong 2Z + \frac{Z^2}{S_f} \cong 2Z + \frac{Z^2}{157.069} \tag{4}$$

Here S_f is a new number and can be called as the nuclear stability factor and A_S is stable mass number. With reference to the ratio of neutron and electron rest masses, S_f can be expressed

$$S_f \cong \sqrt{\alpha} \cdot \frac{m_n}{m_e} \cong 157.0687113 \tag{5}$$

Here α is the fine structure ratio. If Z= 21, A_S = 44.8, Z= 29, A_S = 63.35, Z=47, A_S = 108.06, Z=79, A_S = 197.73 and Z=92, A_S = 237.88. By considering A as the fundamental input its corresponding stable Z = Z_S takes the following form.

$$Z_S \cong \left[\sqrt{\frac{A}{157.069} + 1} - 1\right] 157.069 \tag{6}$$

Thus Green's stability formula in terms of Z takes the following form.

$$\frac{0.4A^2}{A + 200} \cong A_S - 2Z \cong \frac{Z^2}{S_f}. \tag{7}$$

Surprisingly it is noticed that this number S_f plays a crucial role in fitting the nucleons rest mass. Another interesting observation is that

$$(m_n - m_p)\, c^2 \cong \ln\left(\sqrt{S_f}\right) m_e c^2 \cong 1.29198 \text{ MeV} \tag{8}$$

Here m_n, m_p and m_e are the rest masses of neutron, proton and electron respectively. Semi empirically it is noticed that

$$\frac{E_c}{2E_a} \cdot \frac{e^{S_f}}{N} \cong \frac{e^2}{4\pi\varepsilon_0 G m_e^2} \tag{9}$$

Electron rest mass can be expressed as

$$m_e \cong \sqrt{\frac{2E_a}{E_c} \cdot \frac{N}{e^{S_f}}} \cdot \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}} \tag{10}$$

Here N is the Avogadro number. $\frac{e^2}{4\pi\varepsilon_0 G m_e^2}$ is the electromagnetic and gravitational force ratio of electron. In this proposal the important questions are: What is the role of Avogadro number in β decay ? and How to interpret the expression $\sqrt{\frac{e^2}{4\pi\varepsilon_0 G}}$? This is a multi-purpose expression. Either the value of Avogadro number or the value of gravitational constant can be fitted. From the semi empirical mass formula if E_a= 23.21 MeV and E_c= 0.71 MeV,

$$G \cong \frac{2E_a}{E_c} \cdot \frac{N}{e^{S_f}} \cdot \frac{e^2}{4\pi\varepsilon_0 m_e^2} \cong 6.6866323 \times 10^{-11} \ \mathrm{m^3 Kg^{-1} sec^{-2}} \tag{11}$$

Since all other atomic constants are well measured, accuracy of G only depends upon E_a and E_c of the semi empirical mass formula. Multiplying and dividing RHS of equation (10) by N

$$m_e c^2 \cong \sqrt{\frac{2E_a}{E_c} \cdot \frac{N^3}{e^{S_f}}} \cdot \sqrt{\frac{e^2}{4\pi\varepsilon_0} \cdot \frac{c^4}{N^2 G}} \cong X_E \cdot \sqrt{\frac{e^2}{4\pi\varepsilon_0} \cdot \frac{c^4}{N^2 G}} \tag{12}$$

where $X_E \cong \sqrt{\frac{2E_a}{E_c} \cdot \frac{N^3}{e^{S_f}}}$ can be called as the 'gravitational mass generator' of electron.

$$X_E \cong \sqrt{\frac{2E_a}{E_c} \cdot \frac{N^3}{e^{S_f}}} \cong N\sqrt{\frac{4\pi\varepsilon_0 G m_e^2}{e^2}} \cong 295.0606338 \tag{13}$$

$$\sqrt{\frac{e^2}{4\pi\varepsilon_0 N^2 G}} \cong 3.087291597 \times 10^{-33} \ \mathrm{Kg} \tag{14}$$

$$\frac{c^4}{N^2 G} \cong 3.337152088 \times 10^{-4} \ \mathrm{newton} \tag{15}$$

$$\sqrt{\frac{e^2 c^4}{4\pi\varepsilon_0 N^2 G}} \cong \sqrt{\frac{e^2}{4\pi\varepsilon_0} \cdot \frac{c^4}{N^2 G}} \cong 1.731843735 \times 10^{-3} \ \mathrm{MeV} \tag{16}$$

4 The beginning of Strong nuclear gravity

1. For any one elementary particle of mass m_0, magnitude of gravitational constant is G only. As the number of particles increases to Avogadro number (N), magnitude of gravitational constant approaches $N.G$. Mass of the system approaches to $N.m_0$. This idea leads to "nuclear strong gravity" in the following way.

2. Based on strong gravity, similar to the 'Schwarzschild radius', size of the system can be expressed as $R_N \cong \frac{2(NG)(Nm_0)}{c^2}$. Volume of one particle can be expressed as total volume divided by Avogadro number = $\frac{4\pi}{3N} R_N^3$.

3. Similar to the classical force limit $\frac{c^4}{G}$, force required to bind N particles can be expressed as $\frac{c^4}{(NG)}$. Force required to bind one particle is $\frac{1}{N} \cdot \frac{c^4}{NG} \cong$

$\frac{c^4}{N^2G}$. Rest energy of electron is very close to $m_e c^2 \cong 8\pi\sqrt{\hbar c \cdot \frac{c^4}{N^2G}} \cong \frac{8\pi}{\sqrt{\alpha}}\sqrt{\frac{e^2}{4\pi\varepsilon_0}\cdot\frac{c^4}{N^2G}} \cong 0.50952547$ MeV.

4. If nuclear charge radius or characteristic size of nucleus is$R_0 \approx 1.20$ fm, its volume $V_0 \cong \frac{4\pi}{3}R_0^3$ and total volume of N nucleons is $V_N \cong N \cdot \frac{4\pi}{3}R_0^3$. Thus size of N nucleons is $R_N \cong N^{\frac{1}{3}}R_0 \cong \frac{2(NG)(Nm_0)}{c^2}$. Then rest energy of nucleon comes out to be $m_0c^2 \approx 105\ MeV$. This is not matching with the rest energy of nucleon but matching with the geometric mean of nucleon rest energy and its pairing energy constant, 12 MeV. If α_s is the strong coupling constant, it is noticed that $\frac{1}{\alpha_s}m_0c^2 \approx 939\ MeV$ and $\alpha_s \cdot m_0c^2 \approx 12\ MeV$. More over it is noticed that $m_0c^2 \approx 105\ MeV$ is close to "half of the QCD energy scale" $\approx 217\ MeV$. It is also noticed that $\ln\sqrt{\frac{c^4}{N^2G} \div \frac{e^2}{4\pi\varepsilon_o R_N^2}} \cong \frac{1}{\sqrt{\alpha}}$.

5. Considering m_0 as a characteristic strong interaction energy constant it is noticed that $\sqrt{e^{\frac{m_0}{m_e}}} \times \frac{Gm_e^2}{c} \cong e^{\frac{m_0}{2m_e}} \times \frac{Gm_e^2}{c} \cong \hbar$. Here m_e is the rest mass of electron. If so it can be represented as $\frac{m_0}{m_e} \cong \ln\left(\frac{\hbar c}{Gm_e^2}\right)^2 \cong 206.1113643$ and $m_0c^2 \cong 105.3226825\ MeV$.

6. Considering strong interaction, inverse of the 'strong coupling constant' can be expressed as $\frac{1}{\alpha_s} \approx \ln\sqrt{\frac{2(G)(Nm_0)}{c^2} \div \frac{\hbar}{(Nm_0)c}} \cong 8.9020493 \cong \frac{1}{0.112334} \cong X_S$. Interpretation seems to be $\frac{2(G)(Nm_0)}{c^2}$ is the black hole radius of (Nm_0) in free space and $\frac{\hbar}{(Nm_0)c}$ is the Compton length of (Nm_0). Natural logarithm of square root of ratio of these two lengths represents the inverse of the 'strong coupling constant'.

7. Another fit is $R_0 \cong \left(\frac{m_e}{m_0}\right)^3 \cdot \frac{NG\sqrt{(Nm_p)(Nm_e)}}{c^2} \cong \left(\frac{m_e}{m_0}\right)^3 \cdot \left(\frac{N^2G}{c^4}\right) \cdot \sqrt{m_p m_e}\ c^2 \cong$ 1.2 fm where m_p is the rest mass of proton.

8. It also noticed that $R_0 \cong \frac{1}{N^2} \cdot \left(\frac{\hbar c}{Gm_e^2}\right)^2 \cdot \frac{2Gm_e}{c^2} \cong \frac{e^{\frac{m_0}{m_e}}}{N^2} \cdot \frac{2Gm_e}{c^2}$. It can be expressed as $\hbar \cong m_e\sqrt{\frac{(NG)(Nm_e)R_0}{2}}$ or $\frac{\hbar}{m_e} \cong \sqrt{\frac{(NG)(Nm_e)R_0}{2}}$. Whether it is real or imaginary, this is a very strange and interesting relation.

9. Thus it is noticed that, $R_0 \cong \sqrt{\frac{\sqrt{m_p m_e}}{m_0}} \cdot \frac{\sqrt{2}\hbar}{m_0 c} \cong 1.2081$ fm. Interesting observation is $\sqrt{\frac{\sqrt{m_p m_e}}{m_0}} \cdot \frac{\hbar}{m_0 c} \cong 0.8543$ fm. This can be compared with the 'rms' radius of proton. It can be suggested that, in nuclear electron scattering experiments minimum distance between proton and electron is close to $\sqrt{2}$ times the proton size.

10. Considering the rest mass of electron, its gravitational mass generator can be expressed as $X_E \cong m_e c^2 \div \sqrt{\frac{e^2}{4\pi\varepsilon_0}\left(\frac{c^4}{N^2G}\right)} \cong 295.0606338$. Using this number, tau and muon masses can be fitted accurately.

11. $\sqrt{\frac{e^2c^4}{4\pi\epsilon_0 N^2 G}} \cong 0.0017318\ MeV$ can be considered as the basic electromagnetic mass unit. It can be compared with the characteristic mass unit of 'dark matter'.

12. Product (αX_E) can be considered as the ratio of down and up quark masses and can also be considered as the 'inverse' of the weak coupling angle. Inverse of the 'strong coupling constant' can also be expressed as $\frac{1}{\alpha_s} \cong \ln\left(X_E^2\sqrt{\alpha}\right) \cong 8.91424 \cong X_S$. It is also noticed that in Hydrogen atom $\frac{a_0}{R_0} \cong \frac{X_E^2}{2}$ where a_0 is the Bohr radius of electron.

13. Total energy of electron in Hydrogen atom is $\left(\frac{\alpha^2}{2}\right) m_e c^2 \cong G_F \left(\frac{\hbar}{m_0 c}\right)^{-3}$ where G_F is the Fermi's weak coupling constant. Thus the famous Fermi's weak coupling constant can be expressed as $G_F \cong \left(\frac{\alpha^2}{2}\right) m_e c^2 \left(\frac{\hbar}{m_0 c}\right)^3 \cong \left(\frac{m_e}{m_0}\right)\left(\frac{e^2}{4\pi\varepsilon_o m_0 c^2}\right)^2 \left(\frac{\hbar c}{2}\right) \approx \frac{1}{3}\left(\frac{e^2}{4\pi\varepsilon_o m_0 c^2}\right)^3 m_0 c^2$.

14. For any black hole if its density approaches the nuclear mass density its mass approaches to $\mathrm{M_F} \cong \left(\frac{\hbar c}{(NG)m_e^2}\right)^3 \cdot \sqrt{\frac{m_e^3}{m_n}} \cong 1.81 \times 10^{31}$ Kg and can be called as the **Fermi black hole mass limit**. Ratio of **Fermi black hole mass limit** and **Chandrasekhar's mass limit** is 2π.

15. Strong interaction range can be expressed as $b \cong \frac{M_E}{M_P} \cdot \frac{2Gm_n}{c^2} \cong 2.06$ fm where m_n is the average rest mass of nucleon and M_P is planck mass.

5 Proposed new concepts and semi empirical results in Strong nuclear gravity

1. N being the Avogadro number, mole number of particles effective atomic gravitational constant G_A is equal to N times the classical gravitational constant G.

2. In mole particles, nuclear weak force magnitude for one one particle is $F_W \cong \frac{c^4}{NG_A} \cong 3.337152088 \times 10^{-4}$ newton. Nuclear strong force and weak force magnitudes can be correlated as $\sqrt{\frac{F_S}{F_W}} \cong 2\pi \ln\left(N^2\right)$. Thus $F_S \cong 157.9944058$ newton.

3. Fine structure ratio is $\frac{1}{\alpha} \cong \frac{1}{2}\sqrt{X_E^2 - \left[\ln\left(N^2\right)\right]^2} \cong 136.9930484$. It can be expressed as $\frac{1}{\alpha} \cong \frac{1}{2}\sqrt{X_E^2 - \left(\frac{F_S}{4\pi^2 F_W}\right)} \cong 137.036$.

4. Characteristic nuclear size is $R_0 \cong \sqrt{\frac{e^2}{4\pi\varepsilon_0 F_S}} \cong 1.208398568$ fm.

5. Nuclear weak energy constant is $E_W \cong \sqrt{\frac{e^2 F_W}{4\pi\varepsilon_0}} \cong 1.731844 \times 10^{-3}$ MeV.

6. Muon and tau rest energy $= \left(mc^2\right)_n \cong \left[X_E^3 + \left(n^2 X_E\right)^n \sqrt{N}\right]^{\frac{1}{3}} E_W$ where n = 1 and 2. At n = 1, $\left(mc^2\right) \cong 105.95$ MeV, n = 2, $\left(mc^2\right) \cong 1777.4$ MeV. At n = 3 predicted heavy charged lepton rest energy is 42260 MeV.

7. The proton-nucleon nuclear stability factor is $S_f \cong X_E - \frac{1}{\alpha} - 1 \cong 157.0246441$. Proton and nucleon stability relation can be expressed as, stable mass number $= A_S \cong 2Z + \frac{Z^2}{S_f}$ where Z is the proton number.

8. At n = 1 and $n = 2$, with reference to electron rest mass , neutron and proton rest energy $= \left(mc^2\right)_n \cong \frac{S_f}{\sqrt{\alpha}} m_e c^2 - x\left(2^x + \frac{E_c}{2E_a}\right) m_e c^2$ where $x = (-1)^n$, E_c and E_a are the coulombic and asymmetry energy constants of semi empirical mass formula.

9. Nuclear strong energy constant is $E_S \cong \sqrt{\frac{e^2 F_S}{4\pi\varepsilon_0}} \cong 1.191630355$ MeV and nuclear coulombic energy constant is $E_c \cong \frac{3}{5} E_S \cong 0.715$ MeV.

10. Proton rest mass is $m_p c^2 \cong \left(\frac{F_S}{F_W} + X_E^2 - \frac{1}{\alpha^2}\right) E_W \cong 938.1791391$ MeV. Neutron, proton mass difference is $m_n c^2 - m_p c^2 \cong \sqrt{\frac{F_S}{F_W} + X_E^2} \cdot E_W \cong 1.29657348$ MeV.

11. Electroweak energy scale is $\varepsilon_W \cong \frac{F_S}{F_W} \times m_e c^2 \cong \frac{F_S}{F_W} \times 0.511 \cong 241927.75$ MeV. This is a very simple confirmation for the proposed definitions of F_S and F_W.

12. Weak coupling angle is $\sin\theta_W \cong \frac{1}{\alpha X_E} \cong 0.464433353 \cong \frac{\text{Up quark mass}}{\text{Down quark mass}}$.

13. Relation between electron rest mass and up quark rest mass can be expressed as $\frac{Uc^2}{m_e c^2} \cong \left[\frac{m_e^2 c^4}{\hbar c F_W}\right]^{\frac{1}{3}} \cong 8.596650881 \cong e^{\alpha X_E}$. Relation between up quark and down quark rest masses is $\frac{Dc^2}{Uc^2} \cong \ln\left[\frac{Uc^2}{m_e c^2}\right] \cong 2.151372695 \cong \alpha X_E$. Up, strange and bottom quarks are in first geometric series and Down, charm and top quarks are in second geometric series.

14. USB geometric ratio is $g_U \cong \left[\frac{D}{U} \cdot \frac{D+U}{D-U}\right]^2 \cong \left[\alpha X_E \cdot \frac{\alpha X_E + 1}{\alpha X_E - 1}\right]^2 \cong 34.66$ and DCT geometric ratio is $g_D \cong \left[2 \cdot \frac{D}{U} \cdot \frac{D+U}{D-U}\right]^2 \cong \left[2 \cdot \alpha X_E \cdot \frac{\alpha X_E + 1}{\alpha X_E - 1}\right]^2 \cong 138.64 \cong 4 r_U$.

15. Surprisingly it is also noticed that $\frac{1}{\alpha_s} \cong \ln\left(r_U r_D\right) \cong 8.4747 \cong \frac{1}{0.1179598}$. Interesting observation is $\left(\frac{1}{\alpha} + \frac{1}{\alpha_s}\right)\sqrt{UD} \cdot c^2 \cong m_p c^2$ and $\frac{\sqrt{UD} \cdot c^2}{(m_n - m_p)c^2} \cong \ln\left(\frac{1}{\alpha} + \frac{1}{\alpha_s}\right)$ where m_p and m_n are the rest mass of proton and neutron.

16. Magnetic moment of electron is $\mu_B \cong \frac{ec}{2}\sqrt{\frac{e^2}{4\pi\varepsilon_0 F_W}}\sin\theta_W$ and magnetic moment of nucleon is $\mu_n \cong \frac{ec}{2}\sqrt{\frac{e^2}{4\pi\varepsilon_0 F_S}}\sin\theta_W \cong \frac{ecR_0}{2}\sin\theta_W$ where R_0 is unit nuclear size or nucleon size.

17. If nucleon's magnetic moment follows the expression, $\mu \cong \frac{ecR_0}{2} sin\theta_W$, it is noticed that for proton and neutron, $R_0 \cong e^{X_E}\left(\frac{Gm_e^2}{\hbar c}\right)^2 \frac{2Gm_n}{c^2} \pm \frac{\hbar}{m_n c} \cong$ 1.27 fm and 0.85 fm where $e^{X_E}\left(\frac{Gm_e^2}{\hbar c}\right)^2 \frac{2Gm_n}{c^2} \cong 1.06$ fm.

18. Total energy of electron in hydrogen atom can be expressed as $\frac{e^2}{8\pi\varepsilon_0 a_0} \cong \frac{1}{2}\alpha^2 m_e c^2 \cong \frac{1}{X_E^2}\times\sqrt{\frac{e^2 F_S}{4\pi\varepsilon_0}} \cong \frac{1}{X_E^2}\times\frac{e^2}{4\pi\varepsilon_0 R_0}$. Here a_0 is the Bohr radius and R_0 is the nuclear characteristic size. Thus it is noticed that $\hbar \cong N\sqrt{\frac{Gm_e^3 R_0}{2}}$ and $\frac{a_0}{R_0} \cong \frac{X_E^2}{2}$.

19. $X_S \cong \ln\left(X_E^2\sqrt{\alpha}\right) \cong 8.91424 \cong \frac{1}{\alpha_s}$ can be considered as 'inverse of the strong coupling constant'.

20. With reference to proton rest energy, semi empirical mass formula coulombic energy constant can be expressed as $E_c \cong \frac{\alpha}{X_S}\cdot m_p c^2 \cong \alpha\cdot\alpha_s\cdot m_p c^2 \cong 0.7681\ MeV$.

21. Pairing energy constant is close to $E_p \cong \frac{m_p c^2 + m_n c^2}{S_f} \cong 11.959\ MeV$ and asymmetry energy constant can be expressed as $E_a \cong 2E_p \cong 23.918\ MeV$.

22. Volume and surface energy constants and asymmetric and pairing energy constants can be related as $E_a - E_v \cong E_s - E_p \cong (X_S + 1)E_c \cong 7.615\ MeV$. $E_v + E_s \cong E_a + E_p \cong 3E_p$. Thus $E_v \cong 16.303$ MeV and $E_s \cong 19.574\ MeV$.

23. It is also noticed that, $\frac{E_a}{E_v} \cong 1 + \sin\theta_W$ and $\frac{E_a}{E_s} \cong 1 + \sin^2\theta_W$. Thus $E_v \cong 16.332$ MeV and $E_s \cong 19.674\ MeV$.

24. Nuclear binding energy can be fitted with 2 terms or 5 factors with $E_c \cong 0.7681$ MeV as the single energy constant. First term $= T_1 \cong (f)(A+1)\ln[(A+1)X_S]E_c$, second term $= T_2 \cong \left[\frac{A^2 + (f.Z^2)}{X_S^2}\right]E_c$ where $f \cong 1 + \frac{2Z}{A_S} \cong \frac{4S_f + Z}{2S_f + Z} < 2$ and $A_S \cong 2Z + \frac{Z^2}{S_f} \cong 2Z + \frac{Z^2}{157.025}$. Close to the stable mass number, binding energy $= T_1 - T_2$.

6 Basic concepts and semi empirical results in 'modified' super symmetry

1. There exists a strongly interacting confined fermion of rest energy $M_{Sf}c^2 \cong m_o c^2 \equiv 105.3226825$ MeV. Its boson rest energy can be expressed as

$M_{Sb}c^2 \cong \frac{M_{Sf}c^2}{\Psi} \cong 46.6$ MeV where $\Psi \approx 2.26$ is the fermion and boson mass ratio. In particle physics these mass units play a very interesting role.

2. In SUSY it is assumed that $m_e \cong \alpha.\sqrt{M_{Sf}.M_{Sb}} \cong \alpha.\sqrt{M_{Sf}.\left(\frac{M_{Sf}}{\Psi}\right)}$. In this way value of $\Psi \cong 2.262218404$ is fitted. If m_f and m_b are the rest masses of fermion and boson, $m_b \cong \frac{m_f}{\Psi}$. Interesting thing is that $\left(1-\frac{1}{\Psi}\right)m_f$ acts as the effective fermion.

3. For any massive particle, $\frac{\text{mass}}{X_E} \cong \frac{\text{mass}}{295.0606338}$ can be called its electromagnetic mass. Fine structure ratio $= \alpha^2 \cong \frac{e^2 F_W}{4\pi\varepsilon_0}\left(\frac{M_{Sf}c^2}{X_E}\right)^{-1}\left(\frac{M_{Sb}c^2}{X_E}\right)^{-1}$.

4. Fermi's weak coupling constant $= G_F \cong \frac{1}{2\sqrt{\Psi}}\left(\frac{e^2}{4\pi\varepsilon_0 M_{Sf}c^2}\right)^3 M_{Sf}c^2 \cong \frac{1}{2}\left(\frac{e^2}{4\pi\varepsilon_0 M_{Sf}c^2}\right)^3 \sqrt{M_{Sf}M_{Sb}}\cdot c^2 \cong 1.43358632 \times 10^{-62}$ Jm^3.

5. Characteristic nuclear fermion is $X_S.M_{Sf}c^2 \cong 938.8716604$ MeV and its corresponding nuclear boson is $M_{Sb}c^2 \cong \frac{X_S M_{Sf}c^2}{\Psi} \cong \frac{938.8716604}{\Psi} \cong 415.0225543$ MeV. This boson is the mother of presently believed strange mesons like 493, 548, 1020 MeV etc.

6. There exists Higgs charged fermion of rest energy $M_{Hf}c^2 \cong 103125.417$ MeV. Its corresponding Higgs charged boson rest energy is $M_{Hb}c^2 \cong \frac{M_{Hf}c^2}{\Psi} \cong 45585.97\ MeV$. With reference to Beta decay, it is noticed that $\frac{m_e c^2}{F_W R_0} \cong \frac{M_{Hf}}{m_e} \cong \frac{1}{2}\left(\frac{m_e^2 c^4}{\hbar c F_W}\right)^2$.

7. Higgs charged boson pair generates the electro weak neutral Z boson. Top quark boson is nothing but the SUSY electroweak W boson. Higgs charged boson and W boson couples together to form a neutral boson of rest energy 126 GeV. W boson pair generates a neutral boson of rest energy 161 GeV.

8. **If a charged quark flavour rests in a fermionic container it is a quark fermion. Similarly if a charged quark flavour rests in a bosonic container it is a quark boson. Strong interaction charge contains multiple flavours' and can be called as the hybrid (charge) quark. No three quark fermions couples together to form a baryon and no two quark fermions couples together to form a meson.**

9. There exists nature friendly **integral charge quark fermions** and **integral charge quark bosons**. If Q_f and Q_b are the rest masses of quark fermion and quark boson respectively, $Q_b \cong \frac{Q_f}{\Psi}$. Interesting thing is that $Q_{ef} \cong \left(1-\frac{1}{\Psi}\right)Q_f \cong Q_f - Q_b$ acts as the effective fermion.

10. There exists integral charged massive quark fermi-gluons and integral charged massive quark boso-gluons. Fermi-gluon means massive gluons

Quark	Q_f MeV	Q_F MeV	Q_{ef} MeV	Q_E MeV
Up	4.4	**833.973**	2.455	686.571
Down	9.4755	**1076.966**	5.287	886.615
Strange	152.5427	2719.35	85.11	2238.71
Charm	1313.796	5574.13	733.04	4588.92
Bottom	5287.579	8866.525	2950.24	7299.393
Top	182160.18	28850.43	101637.37	23751.20

Table 1: Proposed quark fermion family rest energies.

Quark	Q_b MeV	Q_B MeV
Up	1.945	368.65
Down	4.189	476.07
Strange	67.43	1202.07
Charm	580.756	2464.01
Bottom	2337.34	3919.39
Top	**80522.81**	12753.16

Table 2: Proposed quark boson and quark meson rest energies.

having fermion behaviour and boso-gluon means massive gluons having boson behaviour. Quark femi-gluon can be called as the 'quark baryon' and quark boso-gluon can be called as 'quark meson'.

11. Quark fermions convert into quark baryons and effective quark fermions convert into effective quark baryons. Similarly quark bosons convert into quark mesons. Effective quark baryons generates charged and unstable multi flavour baryons.'**Integral charge light quark bosons**' in one or two numbers couples with the ground or excited **effective quark baryons** and generates **doublets and triplets**. This is just like 'absorption of photons by the electron'. Please see tables 1 and 2 for the proposed 'quark fermion family' and 'quark boson family' rest energies.

12. Quark baryon rest energy $= Q_F c^2 \cong \frac{\sin\theta_W}{2}\left[M_{Hf}^2 \times Q_f\right]^{\frac{1}{3}} c^2$ and Quark boso-gluon or quark meson rest energy $= Q_B c^2 \cong \frac{\sin\theta_W}{2}\left[M_{Hb}^2 \times Q_b\right]^{\frac{1}{3}} c^2$. Accuracy point of view $\frac{\sin\theta_W}{2}$ can be replaced with $\frac{1}{2\alpha(X_E+1)}$.

13. Rest energy of nucleon is close to $\left(\frac{2U_F D_F}{U_F+D_F}\right) c^2 \cong 940.02$ MeV and nucleon rest energy difference is close to $(m_n - m_p)\, c^2 \cong \sin^2\theta_W \cdot \left(\frac{2U_f D_f}{U_f+D_f}\right) c^2 \cong$ 1.29623 MeV.

14. Effective quark baryon rest energy $= Q_E c^2 \cong \frac{\sin\theta_W}{2}\left[M_{Hf}^2 \times Q_{ef}\right]^{\frac{1}{3}} c^2$. These effective quark baryons play a vital role in fitting the unstable

baryon masses. Quark meson masses play a vital role in fitting the unstable meson masses.

15. Charged ground state baryon rest energy is close to $(Q_{E1}Q_{E2})^{\frac{1}{2}}c^2$ or $(Q_{E1}Q_{E2}^2)^{\frac{1}{3}}c^2$ or $(Q_{E1}Q_{E2}Q_{E3})^{\frac{1}{3}}c^2$ where Q_{E1}, Q_{E2}, and Q_{E3} represents any three effective quark baryons.

16. Neutral ground state meson rest energy is close to $(Q_{B1}+Q_{B2})c^2$ where Q_{B1} and Q_{B2} represents any two quark mesons.

17. Fine rotational levels of any ground state energy m_xc^2 can be expressed as, if n =1,2,3, $(mc^2)_I \cong [n(n+1)]^{\frac{1}{4}} m_xc^2 \cong [I]^{\frac{1}{4}} m_xc^2$ and $(mc^2)_{I/2} \cong \left[\frac{n(n+1)}{2}\right]^{\frac{1}{4}} m_xc^2 \cong \left[\frac{I}{2}\right]^{\frac{1}{4}} m_xc^2$. Super fine rotational levels can be expressed as $(mc^2)_I \cong [n(n+1)]^{\frac{1}{12}} m_xc^2 \cong [I]^{\frac{1}{12}} m_xc^2$ and $(mc^2)_{I/2} \cong \left[\frac{n(n+1)}{2}\right]^{\frac{1}{12}} m_xc^2 \cong \left[\frac{I}{2}\right]^{\frac{1}{12}} m_xc^2$.

7 Conclusion

Now a days scientists are vigorously trying to couple the GTR and quantum mechanics. Authors showed many applications in this new direction. Developing a true grand unified theory at 'one go' is not an easy task. In this critical situation, qualitatively proposed semi empirical relations can be given a chance in understanding and developing the grand unified concepts. Here one important and interesting observation is fitting the gravitational constant with the atomic and nuclear physical constants. This can be considered as the beginning of "**strong (nuclear) gravity**". An alternative to the standard model can be developed. Authors request the world science community to kindly look into this new approach.

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